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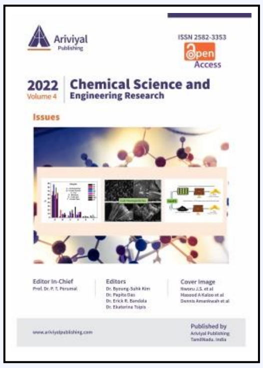

Design of Plug Flow Chemical Reactors with Recycle Flow for Complicated Chemical Reactions by New Approach IAM

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Abstract: One of the important problems in designing plug chemical reactors with recycle flow is the analytical solution of nonlinear chemical reaction integrals. For the design of chemical reactors (especially complicated chemical reactions), the most important task is the analytical solution of mathematical equations, especially integrals and nonlinear differential equations, as a result, we present an innovative method that can easily solve all kinds of complicated integrals at form analytically. In this paper, for the first time, we investigate and solve the complicated highly of chemical reactions integral in the plug flow with the recycle flow for the design of chemical reactors. We challenge and prove the power of this method which can easily analyse for the difficult integral problems which is solved completely analytical way, and we called it *IAM* (Integral Akbari Method) as the following integral for chemical reactions in the reactors:

$$\alpha A_{(g)} \xrightarrow{k} \beta P_{(g)}$$

$$\tau = \frac{V_R C_{A0}}{F_{A0}} = \eta \int_{\lambda 2}^{\lambda 1} \frac{(\gamma 1 + \gamma 2\; x_A) dx_A}{(1 + \varepsilon\; x_A)(1 - x_A)^p}$$

Analytical solution of high integral equations in the design of chemical reactors is very difficult, so we can provide accurate design of chemical reactors by providing the IAM method. We can say with certainty that by presenting this method, there will no longer be any problem for researchers in solving complex integral equations in all fields of engineering, especially chemical engineering and reactor design, as well as in the basic sciences.

Keywords: *New method*; Integral Akbari Method (IAM); Reactor Catalytic bed; Plug flow Reactor; Recycle flow in Reactors

1. Introduction

In this paper we present a new analytical solution in the integral which can easily analyse all such problems and make a great evolution in analytical solution for integrals in the plug flow at industries for the reactors design in chemical engineering fields. Finally, this scientific approach can create a great phenomenon in the analytical solution of integral complicated problems in engineering sciences, especially in the chemical reactions. Our aims introduce of accuracy, capabilities and power for solving complicated integral in the reaction chemical on the plug reactors with recycle flows. IAM method can be successfully applied in various engineering fields such as mechanics (solid and fluid), electronics, petroleum industry, designing chemical reactors,[1,2] and also in applied sciences (physics), economics and so on. It is worth noting that these two methods are convergent at any form of differential equations, including any number of initial and boundary conditions. During the solution procedure, it is not required to convert or simplify the exponential, trigonometric and logarithmic terms, which enables the user to obtain a highly precise solution. Besides, the methodologies behind these techniques are completely understandable, easy to use, and users with common knowledge of mathematics will be capable of solving the most complicated equations at low calculation cost. As all experts know most of engineering actual systems behavior in practical are nonlinear process and analytical scrutiny these nonlinear problems are difficult or sometimes impossible. Our purpose is to enhance the ability of solving the mentioned nonlinear differential equations at chemical engineering and similar issues with a simple and innovative approach which entitled “Integral Akbari Method” or “IAM”. He’s Amplitude Frequency Formulation method[3-5] which was first presented by Ji- Huan He gives convergent successive approximations of the exact solution and Homotopy perturbation technique HPM. It is necessary to mention that the above methods do not have this ability to gain the solution of the presented problem in high precision and accuracy so nonlinear differential equations such as the presented problem in this case study should be solved by utilizing new approaches like AGM[6-12] that created by Mohammadreza Akbari (in 2014). In recent years, analytical methods in solving nonlinear differential equations have been presented and created by Mohammadreza Akbari, these methods are called, AKLM[14] (Akbari Kalantari Leila Method in 2020), ASM[15-17] (Akbari Sara's Method in 2019), AYM[18,19] (Akbari Yasna's Method in 2020), MR.AM (MohammadReza Akbari Method in 2020), and IAM[20] method (Integral Akbari Method). These examples somehow can be considered as complicated cases to deal with for all of the existed analytical methods especially in the design reactor in chemical engineering, which means old methods cannot resolve them precisely or even solve them in a real domain.

2. Mathematical formulation of the problem

Example - 1

Often isomerization reactions are highly two-way (reversible). For example, the isomerization of 1-butene to isobutene is an important step in the production of methyl tertiary butyl ether (MTBE), a common oxygenated additive in gasoline used to lower emissions. MTBE is produced by reacting isobutene with methanol:

$$CH_3 - C(=CH_2) - CH_3 + CH_3OH = CH_3 - (CH_3)C(OCH_3) - CH_3 \tag{1}$$

In order to make isobutene, n-butane (an abundant, cheap C, hydrocarbon) can be dehydrogenated to 1-butene then isomerized to isobutene. Derive an expression for the concentration of isobutene formed as a function of time by the isomerization of 1-butene:

$$CH_2 = CHCH_2CH_3 \underset{K2}{\overset{K1}{\rightleftarrows}} CH_3C(=CH_2)CH_3 \tag{2}$$

Let isobutene be denoted as component I and 1-butene as B If the system is at constant T, then:

$$B \underset{k2}{\overset{k1}{\rightleftarrows}} I \tag{3}$$

By assuming the reaction chemical rate for components B, I is as follows:

$$r_B = k2C_I^q - k1C_B^p \tag{4}$$

The values of p, q are orders of chemical reaction.
The plug flow reactor with recycle flow is shown as (Fig. 1).

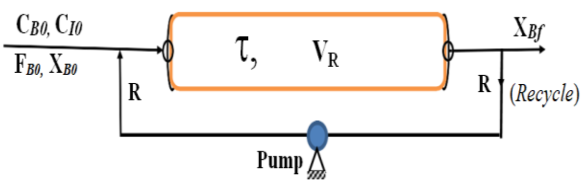

Fig. 1. The schematic system for the plug flow.

Here τ is the reaction time to reach the conversion percentage x_{Bf} and V_R of the reactor volume.
According to the reaction chemical rate Eq. (4), we have:

$$\frac{dC_B}{dt} = k2\, C_I^q - k1\, C_B^p \tag{5}$$

Since,

$$C_B = C_{B0}(1 - x_B) \tag{6}$$

And,

$$C_I = C_{I0} + C_{B0}x_B = C_{B0}(M + x_B)\,, M = \frac{C_{I0}}{C_{B0}} \tag{7}$$

Thus, according to Eq. (5), we have:

$$\frac{dx_B}{dt} = k1(1 - x_B)^p - k2\,(M + x_B)^q \tag{8}$$

At equilibrium:

$$\frac{dx_B}{dt} = 0 \to k_c = \frac{k1}{k2} = \frac{C_{Ieq}^p}{C_{Beq}^q} = \frac{C_{B0}(M + x_{Beq})^q}{C_{B0}(1 - x_{Beq})^q} \tag{9}$$

Insertion of the equilibrium relationship Eq. (9) into the rate expression yields Eq. (8) as follows:

$$\frac{dx_B}{dt} = k1(1 - x_B)^p - k1(1 - x_{Beq})^p \frac{C_{B0}(M + x_B)^q}{C_{B0}(M + x_{Beq})^q} \tag{10}$$

With considering as:

$$\eta = \frac{(1 - x_{Beq})^p}{(M + x_{Beq})^q} \tag{11}$$

According to Eq. (10), we have:

$$\frac{dx_B}{dt} = k1(1 - x_B)^p - \eta(M + x_B)^q \tag{12}$$

For a plug flow (Fig. 1) chemical reactor, the following equation applies to the volume of the chemical reactor:

$$\tau = \frac{V_R\, C_{B0}}{F_{B0}} = -(R + 1)\int_{\beta}^{\alpha} \frac{dc_B}{-r_B} \tag{13}$$

Parameters are as:

$$\alpha = C_{B0}\left(1 - x_{Bf}\right),\ \beta = \frac{C_{B0} + \alpha\, R}{R + 1} \tag{14}$$

$$r_B = -C_{B0}\frac{dx_B}{dt}, dc_B = -C_{B0}dx_B \tag{15}$$

And finally by substituting Eqs. (12,15) into Eq. (13) as follows:

$$\tau = \frac{V_R\, C_{B0}}{F_{B0}} = \lambda \int_{\beta}^{\alpha} \frac{dx_B}{(1 - x)^p - \eta\,.\,(M + x)^q} \tag{16}$$

And also parameter:

$$\lambda = \frac{R+1}{k1} \tag{17}$$

The solution process by new approach IAM (Integral Akbari Method) for complicated integral Eq. (16), the integral analytical solution by IAM as follows:

$$\tau(X_B) = \frac{\lambda(X_B-\beta)}{6}\left\{\Omega + \frac{1}{(1-X_B)^p - \eta\,(M+X_B)^q} + \frac{2}{\left(1-\frac{2\beta}{3}-\frac{X_B}{3}\right)^p - \eta\left(M+\frac{2\beta}{3}+\frac{X_B}{3}\right)^q} + \frac{2}{\left(1-\frac{\beta}{3}-\frac{2X_B}{3}\right)^p - \eta\left(M+\frac{\beta}{3}+\frac{2X_B}{3}\right)^q}\right\} \tag{18}$$

Parameter of Eq. (18) is:

$$\Omega = \frac{1}{(1-\beta)^p - \eta\,(M+\beta)^q} \tag{19}$$

Reaction time (τ) for the percentage of final product X_{Bf} produced in the plug flow reactor as follows:

$$\tau = \frac{\lambda(\alpha-\beta)}{6}\left\{\Omega + \frac{1}{(1-\alpha)^p - \eta\,(M+\alpha)^q} + \frac{2}{\left(1-\frac{2\beta}{3}-\frac{\alpha}{3}\right)^p - \eta\left(M+\frac{2\beta}{3}+\frac{\alpha}{3}\right)^q} + \frac{2}{\left(1-\frac{\beta}{3}-\frac{2\alpha}{3}\right)^p - \eta\left(M+\frac{\beta}{3}+\frac{2\alpha}{3}\right)^q}\right\} \tag{20}$$

We select physical values with metric units as follows:

$$C_{B0} = 0.5\,, C_{I0} = 0.3\,, x_{Bf} = 0.4\,, R = 1.2$$
$$k1 = 1.5\,, p = 1.5\,, q = 1.2\,, x_{Beq} = 0.2 \tag{21}$$

Comparing the achieved solutions by Numerical Method and IAM

Due to the obtained solution from Eq. (18) by IAM and physical values Eq. (21), and Numerical method (Runge-Kutte 4th), we have the following comparisons: (Fig. 2).

The reactor volume diagram in terms of component conversion percentage (B) Shown as Fig. 3.

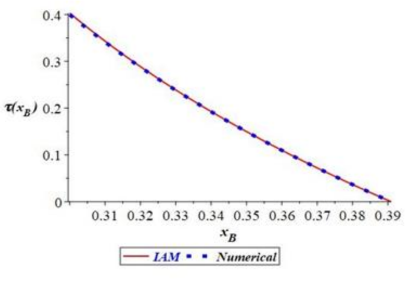

Fig. 2. A comparison between *IAM* and Numerical solution

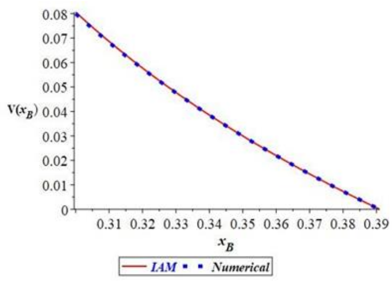

Fig. 3. A comparison between reactor volume IAM and Numerical solution.

According to the physical values Eq. (21), Reaction time (τ) for the percentage of final product X_{Bf} produced in the plug flow reactor as follows

$$\tau_{Numerical} = 0.3998(sec)\,,\ \tau_{IAM} = 0.4031(sec) \tag{22}$$

And the reactor volume of plug flow According to the physical values Eq. (21) as:

$$V_{Reactor} = 0.081(m^3) = 81(liter) \tag{23}$$

The reaction time (τ) in the reactor for different values of the recycle flows (R) in terms of the conversion percentage of component B is given in the following diagram: (Fig. 4).

Figure the reaction time (τ) in the reactor for infinite amount of recycle flow $(R \rightarrow \infty)$ as follows: (Fig. 5).

When the recycle flow tends to infinity$(R \rightarrow \infty)$, the value for the reaction time in the reactor (τ) as well as the reactor volume (V_R) in terms of parameters according to Eq. (20) is obtained as follows.

$$\tau_{(R\rightarrow\infty)} = \frac{\alpha - C_{B0}}{k1\,(1-\alpha)^p - k1\,\eta\,(M+\alpha)^q} \tag{24}$$

The reactor volume as:

$$V_{(R\rightarrow\infty)} = \frac{F_{B0}(\alpha - C_{B0})}{C_{B0}\{k1(1-\alpha)^p - k1\,\eta\,(M+\alpha)^q\}} \tag{25}$$

According to physical parameters as follows:

$$M = \frac{C_{I0}}{C_{B0}}\,,\ \eta = \frac{(1-x_{Beq})^p}{(M+x_{Beq})^q},\ \alpha = C_{B0}(1-x_{Bf}) \tag{26}$$

By substituting Eq. (26) into Eqs. (24,25), we have:

$$\tau_{(R\rightarrow\infty)} = \frac{C_{B0}(1-x_{Bf}) - C_{B0}}{k1\,[1-C_{B0}(1-x_{Bf})]^p - \frac{k1(1-x_{Beq})^p\left[\frac{C_{I0}}{C_{B0}} + C_{B0}(1-x_{Bf})\right]^q}{\left(\frac{C_{I0}}{C_{B0}} + x_{Beq}\right)^q}} \tag{27}$$

And the reactor volume:

$$V_{(R\rightarrow\infty)} = \frac{F_{B0}[C_{B0}(1-x_{Bf}) - C_{B0}]}{C_{B0}k1\left\{[1-C_{B0}(1-x_{Bf})]^p - \frac{(1-x_{Beq})^p\left[\frac{C_{I0}}{C_{B0}} + C_{B0}(1-x_{Bf})\right]^q}{\left(\frac{C_{I0}}{C_{B0}} + x_{Beq}\right)^q}\right\}} \tag{28}$$

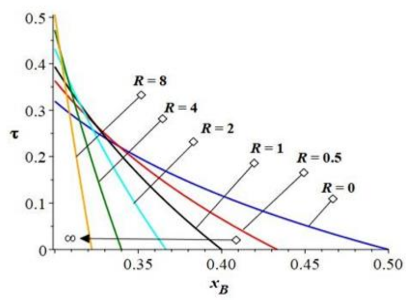

Fig. 4. Reaction time graphs in terms of the values recycle flows (R).

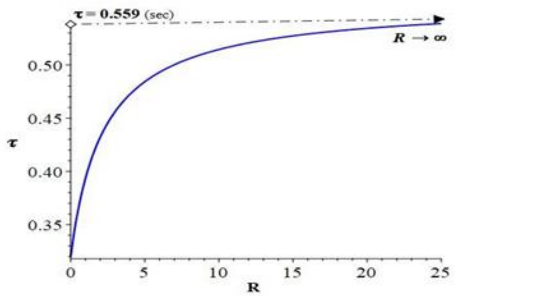

Fig. 5. Reaction time (τ) in the reactor for infinite amount ($R \to \infty$)

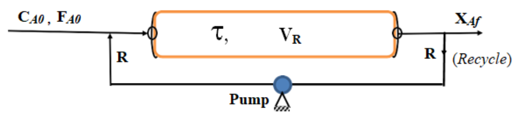

Fig. 6. The schematic system for the plug flow.

Example - 2

We assume chemical reaction at case irreversible and also gaseous state on the solid catalyst bed as follows:

$$\alpha A_{(g)} \xrightarrow{k} \beta P_{(g)} \tag{29}$$

By assuming the reaction chemical rate for component A on the solid catalyst bed as follows:

$$-r_A = \frac{k\, C_A^p}{1 + k\, C_A} \tag{30}$$

The values of p is chemical reaction orders.

The plug flow reactor with recycle flow is shown as (Fig. 6).

Here τ is the reaction time to reach the conversion percentage x_{Af} and V_R of the reactor volume. The equation between the concentration of component A and its conversion percentage is as follows:

$$C_A = C_{A0}\left(\frac{1 - x_A}{1 + \varepsilon\, x_A}\right), \varepsilon = \frac{\beta - \alpha}{\alpha} \tag{31}$$

The plug flow Fig. 2 Chemical reactor, the following equation applies to the volume of the chemical reactor as follows:

$$\tau = \frac{V_R\, C_{A0}}{F_{A0}} = (R+1)\int_{\frac{C_{A0} + R C_{Af}}{R+1}}^{C_{Af}} \left\{\frac{dC_A}{r_A}\right\} \tag{32}$$

Since, derivative of the Eq. (31) as:

$$dC_A = -C_{A0}(1+\varepsilon)\frac{dx_A}{(1 + \varepsilon x_A)^2} \tag{33}$$

By merge Eq. (30), Eq. (32) and Eq. (33) by applying mathematical operations, we have as:

$$\left\{\frac{dC_A}{r_A}\right\} = \frac{-(1+\varepsilon)[1 + \varepsilon\, x_A + k\, C_{A0}(1 - x_A)]dx_A}{k\, C_{A0}^{p-1}(1 - x_A)^p (1 + \varepsilon x_A)} \tag{34}$$

And,

$$C_{Af} = C_{A0}\left(\frac{1 - x_{Af}}{1 + \varepsilon\, x_{Af}}\right) \tag{35}$$

By substituting Eqs. (34, 35) into Eq. (32) by applying mathematical operations and by selecting appropriate parameters, the Eq. (32) is obtained as:

$$\tau = \frac{V_R\, C_{A0}}{F_{A0}} = \eta\int_{\lambda 2}^{\lambda 1} \frac{(\gamma 1 + \gamma 2\, x_A)\, dx_A}{(1 + \varepsilon x_A)(1 - x_A)^p} \tag{36}$$

Parameters Eq. (36) are as:

$$\eta = \frac{-(1+\varepsilon)(R+1)}{k\, C_{A0}^{p-1}},\ \gamma 2 = 1 + k\, C_{A0},\ \gamma 1 = \varepsilon - k\, C_{A0} \tag{37}$$

$$\lambda 1 = C_{A0}\left(\frac{1 - x_{Af}}{1 + \varepsilon\, x_{Af}}\right), \lambda 2 = \frac{C_{A0} + R\,\lambda 1}{R+1}, \varepsilon = \frac{\beta - \alpha}{\alpha} \tag{38}$$

The solution process by IAM for complicated integral Eq. (36), reaction time (τ) for the percentage of final product (x_A) produced in the plug flow reactor as follows: by IAM as follow:

$$\tau(x_A) = \frac{\eta(x_A - \lambda 2)}{6}\left\{\frac{\gamma 2\, x_A + \gamma 1}{(\varepsilon\, x_A + 1)(1 - x_A)^p} + \frac{2\left(\gamma 1 + \gamma 2\left(\frac{2\lambda 2}{3} + \frac{x_A}{3}\right)\right)}{\left(1 + \varepsilon\left(\frac{2\lambda 2}{3} + \frac{x_A}{3}\right)\right)\left(1 - \frac{2\lambda 2}{3} - \frac{x_A}{3}\right)^p} + \frac{2\left(\gamma 2\left(\frac{\lambda 2}{3} + \frac{2x_A}{3}\right) + \gamma 1\right)}{\left(1 + \varepsilon\left(\frac{\lambda 2}{3} + \frac{2x_A}{3}\right)\right)\left(1 - \frac{\lambda 2}{3} - \frac{2x_A}{3}\right)^p}\right\} \tag{39}$$

Parameter of the Eq. (18) is:

$$\Omega = \frac{\gamma 1 + \gamma 2\, \lambda 2}{(\lambda 2\, \varepsilon + 1)(1 - \lambda 2)^p} \tag{40}$$

Reaction time (τ) in the reactor for the percentage of final product x_{Af} produced in the plug flow reactor as follows:

$$\tau = \frac{\eta(\lambda 1 - \lambda 2)}{6}\left\{\frac{\gamma 2\, \lambda 1 + \gamma 1}{(\varepsilon x_A + 1)(1 - \lambda 1)^p} + \frac{2\left(\gamma 1 + \gamma 2\left(\frac{2\lambda 2}{3} + \frac{\lambda 1}{3}\right)\right)}{\left(1 + \varepsilon\left(\frac{2\lambda 2}{3} + \frac{\lambda 1}{3}\right)\right)\left(1 - \frac{2\lambda 2}{3} - \frac{\lambda 1}{3}\right)^p} + \frac{2\left(\gamma 2\left(\frac{\lambda 2}{3} + \frac{2\lambda 1}{3}\right) + \gamma 1\right)}{\left(1 + \varepsilon\left(\frac{\lambda 2}{3} + \frac{2\lambda 1}{3}\right)\right)\left(1 - \frac{\lambda 2}{3} - \frac{2\lambda 1}{3}\right)^p}\right\} \tag{41}$$

We select physical values with metric units as follows:

$$C_{A0} = 0.8, x_{Af} = 0.7, R = 1.2, k = 0.8, p = 1.7, \alpha := 3,$$
$$\beta = 1,\ F_{A0} = 0.5 \tag{42}$$

Comparing the achieved solutions by Numerical Method and IAM

Due to the obtained solution from Eq. (39) by IAM, and Numerical method (Runge-Kutte 4th), we have the following comparisons: (Fig. 7).

The reactor volume diagram in terms of component conversion percentage (A) as follows: (Fig. 8).

According to the physical values Eq. (42), Reaction time (τ) for the percentage of final product (x_{Af}) produced in the plug flow reactor as follows:

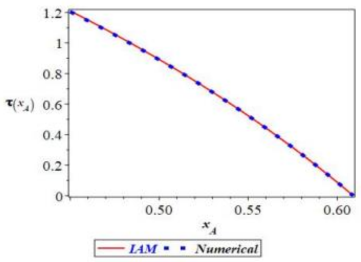

Fig. 7. A comparison between IAM and Numerical solution.

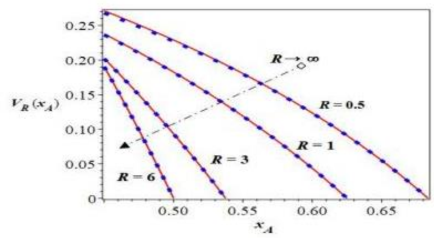

Fig. 9. The reactor volume (V_R) graphs in terms of the values recycle flows (R).

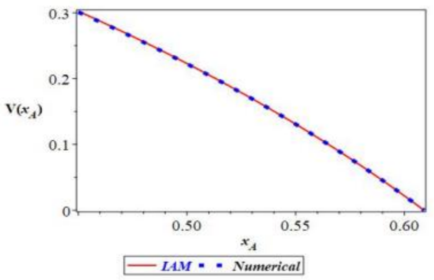

Fig. 8. A comparison between reactor volume IAM and Numerical solution.

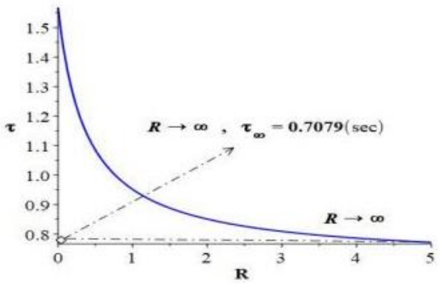

Fig. 10. Reaction time (τ) in the reactor for infinite amount $(R\to\infty)$.

$$\tau_{Numerical} = 0.9172(\text{sec})\,,\ \tau_{IAM} = 0.921(\text{sec}) \tag{43}$$

And reactor volume (V_R) of plug flow according to the physical values Eq. (42) as:

$$V_{Reactor} = 1.15(m^3) \tag{44}$$

The reactor volume (V_R) in the reactor for different values of the recycle flows (R) in terms of the conversion percentage of component A is given in the following diagram (Fig. 9).

Figure the reaction time (τ) in the reactor for amount of recycle flow $(R\to\infty)$ as follows: (Fig. 10).

When the recycle flow tends to infinity $(R\to\infty)$, the value for the reaction time in the reactor (τ) as well as the reactor volume (V_R) is obtained as follows:

$$\tau_{(R\to\infty)} = (-\gamma 2\,\lambda 1^2\,\varepsilon\,C_{A0} + C_{A0}^2\,\gamma 2\,\lambda 1\,\varepsilon - \gamma 1\,\lambda 1\,\varepsilon\,C_{A0} + C_{A0}^2\,\gamma 1\,\varepsilon - C_{A0}\,\gamma 2\,\lambda 1^2 + C_{A0}^2\,\gamma 2\,\lambda 1 - C_{A0}\,\gamma 1\,\lambda 1 + C_{A0}^2\,\gamma 1)/\left(k\,C_{A0}^p(1-\lambda 1)^p\lambda 1\,\varepsilon + (1-\lambda 1)^p\,C_{A0}^p\,k\right) \tag{45}$$

And the reactor volume as:

$$V_{(R\to\infty)} = F_{A0}(-\gamma 2\,\lambda 1^2\,\varepsilon\,C_{A0} + C_{A0}^2\,\gamma 2\,\lambda 1\,\varepsilon - \gamma 1\,\lambda 1\,\varepsilon\,C_{A0} + C_{A0}^2\,\gamma 1\,\varepsilon - C_{A0}\,\gamma 2\,\lambda 1^2 + C_{A0}^2\,\gamma 2\,\lambda 1 - C_{A0}\,\gamma 1\,\lambda 1 + C_{A0}^2\,\gamma 1)/\ C_{A0}\left(k\,C_{A0}^p(1-\lambda 1)^p\,\lambda 1\,\varepsilon + (1-\lambda 1)^p\,C_{A0}^p\,k\right) \tag{46}$$

And the numerical value of the reaction time in the reactor (τ) as well as the volume of the reactor (V_R) for that case $(R\to\infty)$ and according to the physical values Eq. (42) as follows:

$$R\to\infty\,,\ \tau_\infty = 0.7079(sec) \tag{47}$$

$$R\to\infty\,,\ V_\infty \coloneqq 0.4424(m^3) \tag{48}$$

3. Attachment

Application of integrals in the engineering and basic sciences

In this part of the paper, we provide an analytical solution to some examples of complicated and applied integrals by using the IAM method and display them.

Example - 3

We consider a complicated indefinite integral as follows:

$$\int e^{-\beta x^3}\sin\{(\alpha\,x)!\}\,dx \tag{49}$$

Parameter (!) is factorial in mathematics. As we know, this complicated integral can never be solved analytically by common formulas. We solve this integral in the form of analytically by IAM method and then it is calculated numerically in interval [0, 3].

IAM solution process (Integral Akbari's Method)

The output of the complicated integral Eq. (49) analytical solution is obtained by the IAM method process as follows:

$$S(x) = \frac{x}{5}\left\{\frac{1}{2}e^{-\beta x^3}\sin(\alpha\,x)! + \frac{1}{2}\sin(1) + e^{-\frac{\beta x^3}{125}}\sin\left(\frac{\alpha\,x}{5}\right)! + e^{-\frac{8\beta x^3}{125}}\sin\left(\frac{2\alpha\,x}{5}\right)! + e^{-\frac{27\beta x^3}{125}}\sin\left(\frac{3\alpha\,x}{5}\right)! + e^{-\frac{64\beta x^3}{125}}\sin\left(\frac{4\alpha\,x}{5}\right)!\right\} \tag{50}$$

By selecting the physical values for the integral independent parameters with interval [0, 3] as:

$$\alpha = 0.3,\ \beta = 0.2 \tag{51}$$

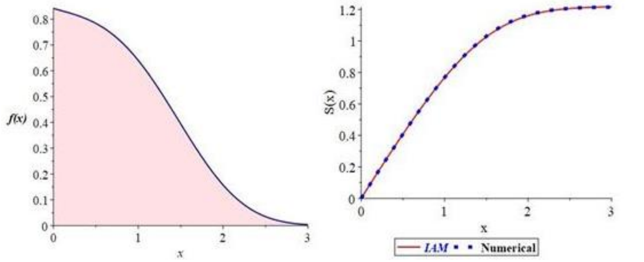

Fig. 11. A comparison between *IAM* method and Numerical solution.

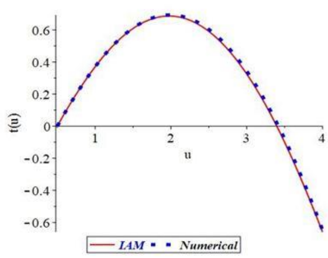

Fig. 12. A comparison between *IAM* method & Numerical solution.

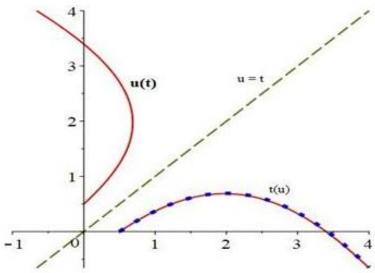

Fig. 13. Comparison between *IAM* method, Numerical solution and inverse answer Function.

Comparing the achieved solutions by IAM and Numerical Method

After analytical solution by IAM method Eq. (50), we solve numerically in interval [0, 3] and according to physical values Eq. (51) as follows: (Fig. 11).

$$\int_0^3 e^{-0.2x^3}\sin\{(0.3\,x)!\}dx \tag{52}$$

Form of numerical solution and analytical solution Eq.(50) are solved in the interval[0, 3], and as well as the maximum error are obtained as follows:

$$S_{Numerical} = 1.214895039, S_{IAM} = 1.2169907$$
$$\Delta_{\text{error}}\% = 0.17 \tag{53}$$

Example - 4

We consider a complicated ***nonlinear differential equation*** as follows:

$$\frac{du}{dt} = \frac{1+\alpha\sin u}{\cos(\beta\, u \sin\sqrt{u})} \tag{54}$$

And initial condition as:

$$ic: u(t) = uo \; at \; t = 0 \tag{55}$$

We rewrite the differential Eq. (54) as follows:

$$\frac{\cos(\beta\, u\sin\sqrt{u})}{1+\alpha\sin u}du = dt \Longrightarrow \int_{uo}^{u}\frac{\cos(\beta\, u\sin\sqrt{u})}{1+\alpha\sin u}du = \int_0^t dt \tag{56}$$

$$t = \int_{uo}^{u}\frac{\cos(\beta\, u\sin\sqrt{u})}{1+\alpha\sin u}du \tag{57}$$

IAM solution process (Integral Akbari's Method)

Answer of the complicated integral Eq. (57) at form analytical solution is obtained by the IAM method process as follows:

$$ta := \frac{u-uo}{6}\left\{\frac{\cos(\beta\, u\sin\sqrt{u})}{1+\alpha\sin u} + \frac{\cos(\beta\, uo\sin\sqrt{uo})}{1+\alpha\sin uo} + \frac{2\cos\left[\beta\left(\frac{2\,uo}{3}+\frac{u}{3}\right)\sin\left(\sqrt{\frac{2\,uo}{3}+\frac{u}{3}}\right)\right]}{1+\alpha\sin\left(\frac{2\,uo}{3}+\frac{u}{3}\right)} + \frac{2\cos\left[\beta\left(\frac{uo}{3}+\frac{2\,u}{3}\right)\sin\left(\sqrt{\frac{uo}{3}+\frac{2\,u}{3}}\right)\right]}{1+\alpha\sin\left(\frac{uo}{3}+\frac{2\,u}{3}\right)}\right\} \tag{58}$$

By selecting the physical values for the integral independent parameters below:

$$\alpha = 0.3, \beta = 0.8, uo = 0.5 \tag{59}$$

Comparing the achieved solutions by ***IAM*** (Integral Akbari's Method) and Numerical Method (Fig. 12).

Comparison of the answer function of the differential Eq. (58) and its inverse solution (Fig. 13).

Example - 5

We consider a complicated ***indefinite*** integral as follows:

$$\underbrace{S(x) = \int\frac{e^{-\alpha x^4}}{1+\beta x^2}dx}_{(analytically)} \; ; \underbrace{S = \int_{-\infty}^{\infty}\frac{e^{-\alpha x^4}}{1+\beta x^2}dx}_{(numerically)} \tag{60}$$

After analytical solution by IAM method, we solve numerically in interval $[-\infty, +\infty]$. Note, the function $f(x)$ is even, so we have:

$$\int_{-\infty}^{\infty} f(x)dx = 2\int_0^{\infty} f(x)dx \tag{61}$$

IAM solution process (Integral Akbari's Method)

The integral analytical solution answer, Eq. (61) is obtained by IAM method as follows:

$$S(x) = \frac{x}{4}\left(1 + \frac{e^{-\alpha x^4}}{(\beta x^2+1)} + \frac{32\, e^{\frac{-\alpha x^4}{256}}}{\beta x^2+16} + \frac{8\, e^{\frac{-\alpha x^4}{16}}}{\beta x^2+4} + \frac{32\, e^{\frac{-81\,\alpha x^4}{256}}}{9\beta x^2+16}\right) \tag{62}$$

By selecting the physical values for the integral independent parameters below:

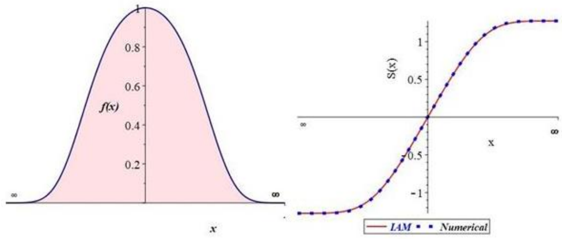

Fig. 14. A comparison between *IAM* method and Numerical solution.

$$\alpha = 2\,, \beta = 1 \tag{63}$$

*Comparing the achieved solutions by **IAM** (Integral Akbari's Method) and Numerical Method* (Fig. 14)

The numerical solution of the integral Eq. (60) in interval $[-\infty, +\infty]$ and according to physical values Eq. (63) is shown below:

$$S = \int_{-\infty}^{\infty} \frac{e^{-2x^4}}{1+x^2} dx \tag{64}$$

Form of numerical solution and analytical solution Eq. (62) are solved in the interval $[-\infty, +\infty]$and as well as the maximum error are obtained as follows:

$$S_{Numerical} = 1.275194143, S_{IAM} = 1.275411544$$
$$\Delta_{\mathrm{error}}\% = 0.017 \tag{65}$$

Example - 6

We consider a complicated indefinite *double* integral as follows:

$$\underbrace{I(x,y) = \int\int e^{-\alpha\sqrt{x^3+y^3}}\,dydx}_{(analytically)};\ \underbrace{\int_0^\infty \int_0^\infty e^{-\alpha\sqrt{x^3+y^3}}\,dydx}_{(numerically)} \tag{66}$$

After analytical solution by *IAM* method, we solve numerically in interval $x \in [0, \infty], y \in [0, \infty]$.

IAM solution process (Integral Akbari's Method)

Form analytically solution by *IAM* method with a very good approximation and so a very low error is obtained as follows:

$$Ixy = \frac{xy}{36}\left\{e^{-\alpha\sqrt{x^3+y^3}} + e^{-\alpha y\sqrt{y}} + 2e^{-\alpha\sqrt{\frac{x^3}{27}+y^3}} + 2e^{-\alpha\sqrt{\frac{8x^3}{27}+y^3}} + \right.$$
$$2e^{-\frac{\sqrt{3}}{9}\alpha x\sqrt{x}} + 2e^{-\frac{2\sqrt{6}}{9}\alpha x\sqrt{x}} + 1 + e^{-\alpha\sqrt{x^3}} + 2e^{-\alpha\sqrt{x^3+\frac{y^3}{27}}} +$$
$$2e^{-\frac{\sqrt{3}\alpha y\sqrt{y}}{9}} + 4e^{-\alpha\sqrt{\frac{x^3}{27}+\frac{y^3}{27}}} + 4e^{-\alpha\sqrt{\frac{8x^3}{27}+\frac{y^3}{27}}} + 2e^{-\alpha\sqrt{x^3+\frac{8y^3}{27}}} +$$
$$\left. 2e^{-\frac{2\alpha\sqrt{6}y\sqrt{y}}{9}} + 4e^{-\alpha\sqrt{\frac{x^3}{27}+\frac{8y^3}{27}}} + 4e^{-\alpha\sqrt{\frac{8x^3}{27}+\frac{8y^3}{27}}}\right\} \tag{67}$$

By selecting the physical values for the integral independent parameters below:

$$\alpha = 0.5 \tag{68}$$

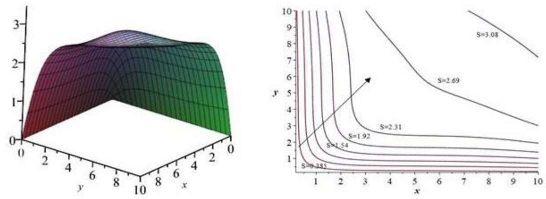

Fig. 15. 3D-diagram and contour diagrams

The 3D-diagram and contour diagrams of achieved solution Eq. (67) by ***IAM*** as follows: (Fig. 15).

The numerical solution of the integral Eq. (66) in interval $x \in [0, \infty], y \in [0, \infty]$ is shown below:

$$Ixy = \int_0^\infty \int_0^\infty e^{-\alpha\sqrt{x^3+y^3}}\,dydx\ ;\ \alpha = 0.5 \tag{69}$$

Numerical solution integral Eq. (69) and analytical solution Eq. (67) are solved in the interval $x \in [0, \infty], y \in [0, \infty]$ and as well as the maximum error are obtained as follows:

$$S_{Numerical} = 2.650155244\,,\ S_{IAM} = 2.658318437$$
$$\Delta_{\mathrm{error}}\% = 0.3 \tag{70}$$

4. Conclusions

In this article, we proved that ***IAM*** can analyse all kinds of complicated practical problems related to integral in the chemical reactors design also which can be easily solved analytically integrals. Obviously, most of the phenomena of chemical reactions in chemical reactors are nonlinear, so it is quite difficult to study and analyse nonlinear mathematical equations in this area, also we wanted to demonstrate the strength, capability and flexibility of the new IAM method (Integral Akbari Method).This method is newly created and it can have high power in analytical solution of all kinds of industrial and practical problems in engineering fields and basic sciences for complicated integral at form analytically.

Acknowledgements

History** of **AGM, ASM, AYM, AKLM, MR.AM and IAM methods:
AGM (Akbari-Ganji Methods), ***ASM*** (Akbari-Sara's Method), ***AYM*** (Akbari-Yasna's Method) ***AKLM*** (Akbari Kalantari Leila Method), ***MR.AM*** (MohammadReza Akbari Method) and ***IAM*** (Integral Akbari Methods), have been invented mainly by Mohammadreza Akbari (M.R.Akbari) in order to provide a good service for researchers who are a pioneer in the field of nonlinear differential equations.

*****AGM*** method Akbari Ganji method has been invented mainly by Mohammadreza Akbari in 2014. Noting that Prof. Davood Domairy Ganji co-operated in this project.

*****ASM*** method (Akbari Sara's Method) has been created by Mohammadreza Akbari on 22 of August, in 2019.

*****AYM*** method (Akbari Yasna's Method) has been created by Mohammadreza Akbari on 12 of April, in 2020.

AKLM method (Akbari Kalantari Leila Method) has been created by Mohammadreza Akbari on 22 of August, in 2020.
MR.AM method (MohammadReza Akbari Method) has been created by Mohammadreza Akbari on 10 of November, in 2020.
IAM method (Integral Akbari Method) has been created by Mohammadreza Akbari on 5 of February, in 2021.

Conflicts of Interest

The authors declare no conflict of interest.

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