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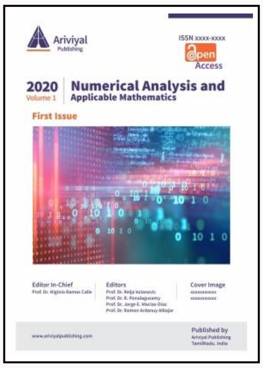

Data-driven Analysis and Prediction of Fractional Order SIR Model for COVID-19

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Abstract: Coronavirus disease (COVID-19) broke out in Wuhan, Hubei province, China, in December 2019. Subsequently, various countries in the world also successively broke out the coronavirus disease. In this paper, we built a mathematical model to predict and analyze the change of COVID-19. Based on the fractional-order susceptible infected-recuperated (SIR) model with the real data from February 29 to March 30, 2020, and the predictor-correctors scheme is applied in this model. Then we use the fitting results and the integer-order ($\alpha = 1$) comparison, the MAD, R^2, BIC and absolute errors of fractional SIR model ($\alpha = 0{:}75$) are smaller than those of integer model. We also predicted the situation of the new case in the next 400 days, the results indicate that the fractional order SIR model have a better fitting and forecasting of the data on the countries China, Italy, Spain, United Kingdom, United states, India and France.

Keywords: Fractional order; SIR epidemic model; Predictor-correctors scheme

1. Introduction

At the end of 2019, the first case of coronavirus disease infection was found in Wuhan, Hubei Province, China. After that, the COVID-19 was found to be able to pass from person to person,[1] and soon other countries around the world also found the infected person. Since then, China has taken strong control measures, emphasizing that people's life safety and physical health should be the first priority, and resolutely curbing the spread of the epidemic, whereas assessing the intervention measures of COVID-19 epidemic poses a major health concern.

Some mathematical models are established to forecast the epidemic of COVID-19. Torrealba-Rodriguez et al.[2] used the logistic and inverse artificial neural network model to predict the number of cases of COVID-19 infection of Mexico. Sun and Wang[3] proposed a modified model to forecast the COVID-19 epidemic in Heilongjiang province of China. Recently, mathematical models of dengue transmission using the fractional order derivative are proposed in Al-Sulami et al. & Derouich et al.[4,5] For example,[6,7] introduced the fractional order susceptible infected-recuperated (SIR) model in dengue transmission. Liu et al.[9] established a new mathematical model for the simulation of the dynamics of a dengue fever outbreak, where the different fractional order α has different simulation effects. In addition, Sahafizadeh et al. & Khan et al.[10,11] transferred fractional derivative to investigate COVID-19 model and gave a reproduction number R_0 of COVID-19 and Podlubny[12] indicated that fractional order model is locally asymptotically stable if $R_0 < 1$.

The application range of fractional order is very wide, and there are many methods to solve its numerical solution. In recent decades, many scholars have pointed out that fractional calculus is very suitable for describing materials and processes with memory and genetic properties, which are often ignored in the classical models.[13,14] A large number of research results show that the numerical discretization of fractional derivative cannot be treated as integer derivative which use one-step arbitrary truncation.[15]

In addition, in the numerical calculation of fractional differential equation, there are explicit scheme, implicit scheme, Crank-Nicholson scheme, predictor-corrector method and integral equation method, which are all finite difference methods. Moreover, some finite element methods, non-network methods and matrix methods are also used in the calculation of fractional differential equations, but the more mature and efficient algorithm is the predictor-corrector method.[16-18] Moreover, some papers about spreading and prediction of COVID-19 were proposed,[19] developed a new mathematical model by including the resistive class together with quarantine class. Sinan et al.[20] used the SEIQR fractional mathematical model to investigate the stability and optimal control of the concerned mathematical model for both local and global stability by third additive compound matrix approach and obtained threshold value by the next generation approach. Next, fractal-fractional derivative is proposed to predict the propagation of COVID-19; the simulation result proves that movement control order has a great impact on the transmission dynamics of the current outbreak in Malaysia in Ali et.al.[21] With help of sensitivity analysis of the reproduction number,[22] found the most sensitive parameters regarding transmission of infection. A mathematical model of Caputo type operator[23] is established to predict the future trend of COVID-19 epidemic trend of confirmed cases and deaths in India in October 2020.

In this paper, we adopt the idea in Okyere et al.[6] and the predictor-corrector method in Ahmed et al.[15] to study the change of COVID-19. We use the fractional SIR epidemic model fitting the real data of China, Italy, Spain, United Kingdom, United States, India and France from February 29 to March 30, 2020. Finally, we predict the possible trend of the epidemic. The results show that the fitting ability of fractional order is better than that of integer order.

2. Predictor-corrector method for fractional differential equations

The predictor-corrector method[15] can better solve the numerical problems of fractional differentiation. The prediction-correction method is more mature than the general rectangle method and has better convergence performance. The following facts will be used in the sequel.
Let $f: [0,\infty] \to \mathbb{R}$ be continuous. Consider the following Cauchy problem for fractional differential equations.[14,16]

$$\begin{cases} CD_{0,t}^{\alpha}\, x(t) = f(x, x(t)), 0 < \alpha \leq 1, t > 0, \\ \qquad x(o) = x_0, \end{cases} \tag{1}$$

where $CD_{0,t}^{\alpha}\, x$ denotes Caputo fractional derivative of x at time t, i.e.,

$$CD_{0,t}^{\alpha}\, x(t) := \frac{1}{\Gamma(1-\alpha)} \int_0^t \frac{x'(s)}{(t-s)^{\alpha}} ds, 0 < \alpha \leq 1, t > 0.$$

Note that (1) can be transferred into the corresponding fractional integral

$$x(t) = x_0 + \frac{1}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} f(s, x(s)) ds. \tag{2}$$

Then one can use the idea of predictor-corrector[15] to design the calculation scheme of integral equation (2). First, the trapezoid formula is used to approximate the integral of formula (2), take step h on integral region $[t_0; t]$, node is $t_j = t_0 + jh$; $(j = 0; 1; \cdots; k+1)$, weight function is $(t_{k+1} - s)^{\alpha-1}$. Use approximation

$$\int_0^{t_{k+1}} (t_{k+1} - s)^{\alpha-1} g(s) ds \approx \int_0^{k+1} (t_{k+1} - s)^{\alpha-1} gk+1(s) ds = \sum_{j=0}^{k+1} a_j, k+1 g(t_j), \tag{3}$$

Where

$$a_j, k+1 = \begin{cases} \frac{h^{\alpha}}{\alpha(\alpha+1)} [k^{\alpha+1} - (k-\alpha)(k+1)^{\alpha}], j = 0 \\ \frac{h^{\alpha}}{\alpha(\alpha+1)} [(k-j+2)^{\alpha+1} - 2(k-j+1)^{\alpha+1} + (k-j)^{\alpha+1}], 1, \leq j \leq k, \\ \frac{h^{\alpha}}{\alpha(\alpha+1)}, j = k+1. \end{cases} \tag{4}$$

Derived correction formula for (2)

$$x_{k+1} = x_0 + \frac{1}{\Gamma(\alpha)} \left(\sum_{j=0}^{k} a_j, k+1 f(t_j, x_j) + a_{k+1}, k+1 f(t_{k+1}, x_{k+1}^{p})\right). \tag{5}$$

where x_{k+1}^{p} is the necessary forecast approximation, the formula (5) is actually a fractional expression of one-step Adams-Moulton prediction-correction formula. The remaining problem is to give the prediction formula, that is, to solve x_{k+1}^{p}, the rectangle formula is given by

$$\int_{t_0}^{t_{k+1}} (t_{k+1} - s)^{\alpha-1} g(s) ds \approx \sum_{j=0}^{k} b_{j,k+1} g(t_j),$$

where $b_{j,k+1} = \frac{h^{\alpha}}{\alpha} [(k+1-j)^{\alpha} - (k-j)^{\alpha}]$. Then we get the estimation formula

$$x_{k+1}^{p} = x_0 + \frac{1}{\Gamma(\alpha)} \sum_{j=0}^{k} b_{j,k+1} f(t_j, x_j). \tag{6}$$

Finally, predictor-corrector method for (1) is presented by (5) and (6).

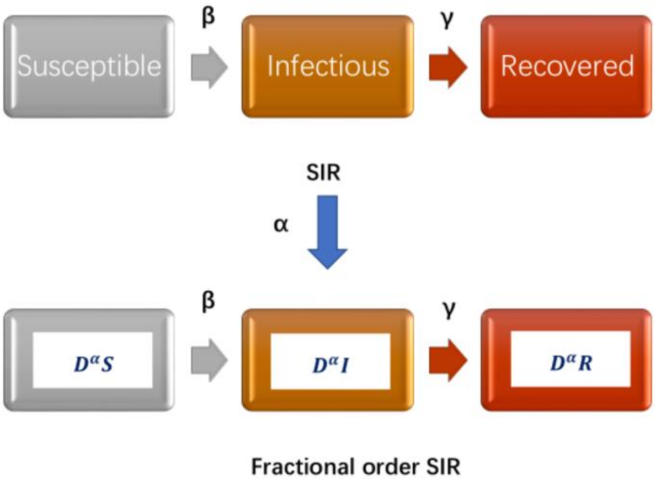

Fig. 1. Schematic illustrations of the fractional order SIR model, where β, γ denote the contact rates and the recovery rate, respectively.

Table 1. Parameter values list.

Parameter	Biological meaning	Value
Λ	the constant supplement rate of the susceptible	0.5
β	contact rates	0.36
μ	natural mortality	0.1
γ	the rate of disease-related death	0.04
ϵ	recovery rate of disease	0.1

3. Fitting and predicted results for fractional order SIR model

Consider the following fractional order SIR model appeared in [6]

$$\begin{cases} {}^{C}D_{0,t}^{\alpha}\, S(t) = \Lambda - \beta S(t)I(t) - \mu S(t), o < \alpha \leq 1, t > 0, \\ {}^{C}D_{0,t}^{\alpha}\, I(t) = \beta S(t)I(t) - (\mu + \gamma + \epsilon)I(t), \\ {}^{C}D_{0,t}^{\alpha}\, R(t) = \gamma I(t) - \mu R(t), \end{cases} \tag{7}$$

where S, I, R represents the number of susceptible, infected and recovered persons at time t, the relationship among three groups of people respectively see Fig. 1; Λ is the constant supplement rate of the susceptible, β is contact rates, μ is natural mortality, ϵ is the rate of disease-related death, γ is recovery rate of disease.

We use predictor-corrector method to deal with (7) and give fitting and predicted results for infect and new cases for COVID-19 in China, Italy, Spain, United Kingdom, United States, India and France. In addition, the average absolute deviation (MAD) and the coefficient of determination (R^2) were used to evaluate the model. The MAD, R^2 are defined as follows:

$$MAD = \frac{\sum_{i=1}^{n} |y_i - \hat{y}_i|}{n} \tag{8}$$

And

$$R^2 = 1 - \frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{\sum_{i=1}^{n}(y_i - \bar{y})^2} \tag{9}$$

We usually use the Bayesian information criterion (BIC) to evaluate the quality of a model. The smaller the BIC value, the better the model.

$$BIC = \log\left(\frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2\right) + \frac{p \log n}{n} \tag{10}$$

3.1. Fitting result

We give the fitting results of fractional order SIR model based on MATLAB (see Fig. 2), we use the parameters value (see Table 1), and the step size h = 0:05.

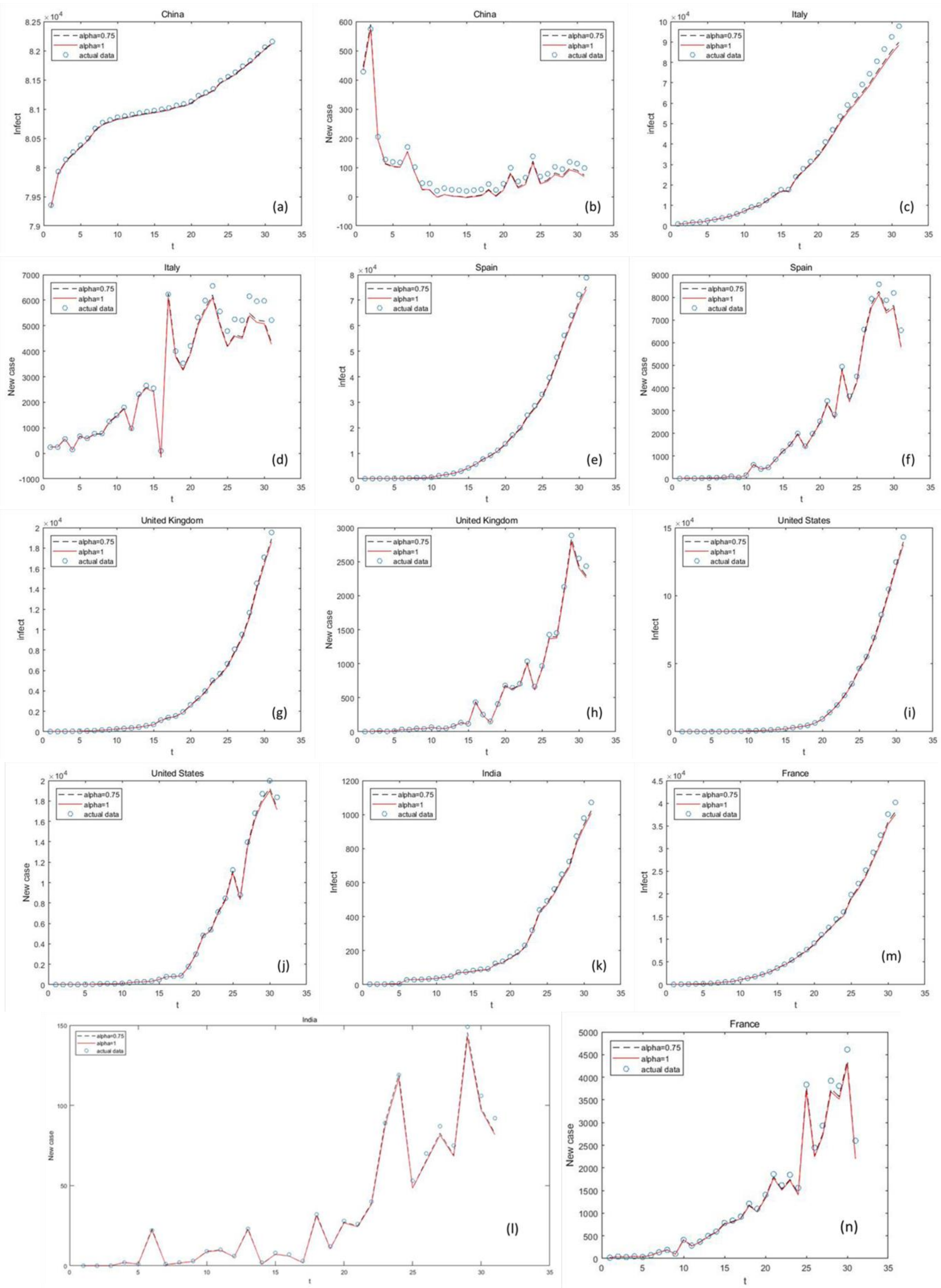

Fig. 2. Fitting results for fractional order SIR model for the seven country: China (a, b), Italy (c, d), Spain (e, f), United Kingdom (g, h), United States (i, j), India (k, l) and France (m, n).

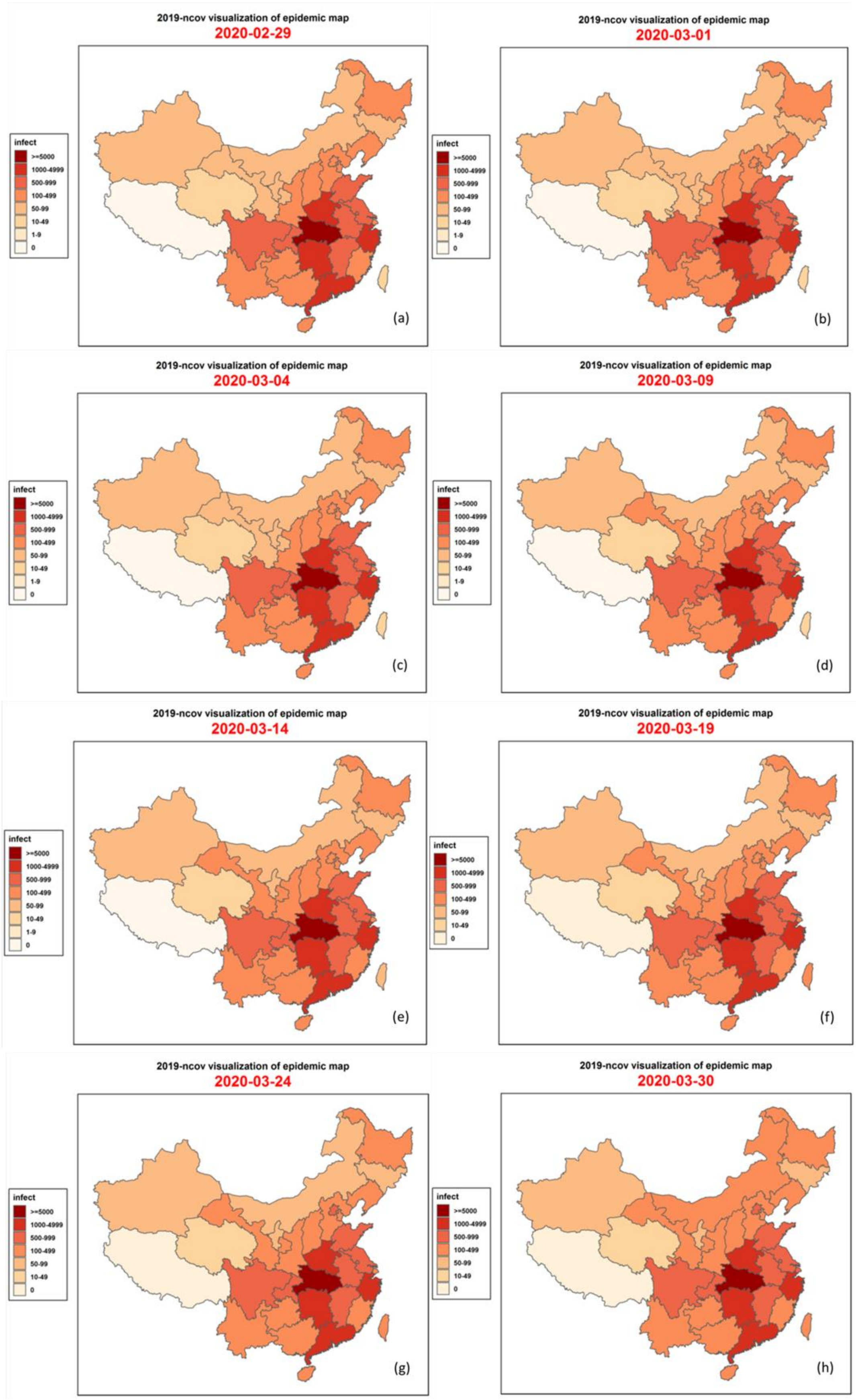

Fig. 3. The visualization maps of the number of infect people of China.

Table 2. Different values of fractional order SIR model

Country	α = 0:75			α = 1		
	MAD	R^2	BIC	MAD	R^2	BIC
China	12445.3066	0.9983	6.5599	12434.5654	0.9967	7.3577
Italy	1704.3700	0.9917	15.9876	2175.7100	0.9876	16.3933
Spain	490.1181	0.9982	13.8902	762.9469	0.9963	14.6241
United Kingdom	90.2936	0.9989	10.4300	152.4867	0.9972	11.3836
United States	322.1497	0.9996	13.5312	738.4315	0.9984	14.8149
India	8.3669	0.9978	5.4643	12.2566	0.9955	6.1721
France	364.2977	0.9969	13.0979	518.0993	0.9944	13.7004

Table 3. Fractional order SIR model for seven countries infect data from March 26 to March 30, 2020.

Country	Date	Real Value	Fitting Value	
			α= 0:75	α = 1
China	3.26	81733	81707.93	81593.28
	3.27	81827	81801.13	81789.10
	3.28	81946	81920.45	81908.53
	3.29	82059	82033.74	82021.25
	3.30	82157	82131.85	82119.97
Italy	3.26	74386	69764.80	68719.43
	3.27	80539	75248.64	74105.32
	3.28	86498	80463.69	79228.51
	3.29	92472	85635.20	84303.86
	3.30	97698	89960.45	88550.19
Spain	3.26	47610	46297.58	45536.28
	3.27	56188	54558.80	53663.12
	3.28	64059	61964.73	60974.03
	3.29	72248	69605.67	68517.05
	3.30	78797	75438.89	74306.46
United Kingdom	3.26	9529	9271.12	9118.46
	3.27	11658	11354.14	11162.34
	3.28	14543	14193.41	13946.68
	3.29	17089	16635.32	16353.47
	3.30	19522	18925.76	18616.22
United States	3.26	69194	68457.97	67260.25
	3.27	85991	84917.94	93443.27
	3.28	104686	103109.53	101349.34
	3.29	124665	122374.30	120335.48
	3.30	143025	139679.46	137448.43
India	3.26	649	628.38	617.88
	3.27	724	697.43	686.23
	3.28	873	842.79	828.69
	3.29	979	941.08	925.84
	3.30	1071	1023.56	1007.62
France	3.26	25233	24221.21	23839.89
	3.27	29155	27966.06	27518.48
	3.28	32964	31539.35	31037.37
	3.29	37575	35901.08	35325.17
	3.30	40174	38104.87	37522.68

In Fig. 2, the left represents the fit of the number of infected people of the countries China, Italy, Spain, United Kingdom, United states, India and France and the right represents the fitting of new cases. It is found that the fitting of fractional order is better that the integral order (α = 1).

According to the fitting results, calculating the values of MAD, R^2 and BIC index in the training sample set (see Table 2). The comparison of fitting value and actual value see Table 3.

We chose the most representative country, China, and made a visual map(Fig. 3), it is shows the change of the number infect people of China, and we can see at this stage February 29 to March 30, 2020, the change of China's epidemic situation is very small, reaching a peak basically. It is similar to our fitting results.

3.2. Predicted result

As is shown in Fig. 4(400 days forecast of new case data after March 30, 2020), as time goes on, the number of new cases predicted by fractional SIR gradually decreases and tends to 0, and with the decrease of α, the time of disease elimination is longer. But new cases such as the United States and India have a very long period of time, and the rate of reducing to zero is very slow, probably because the measures taken are not as strict as those of other countries.

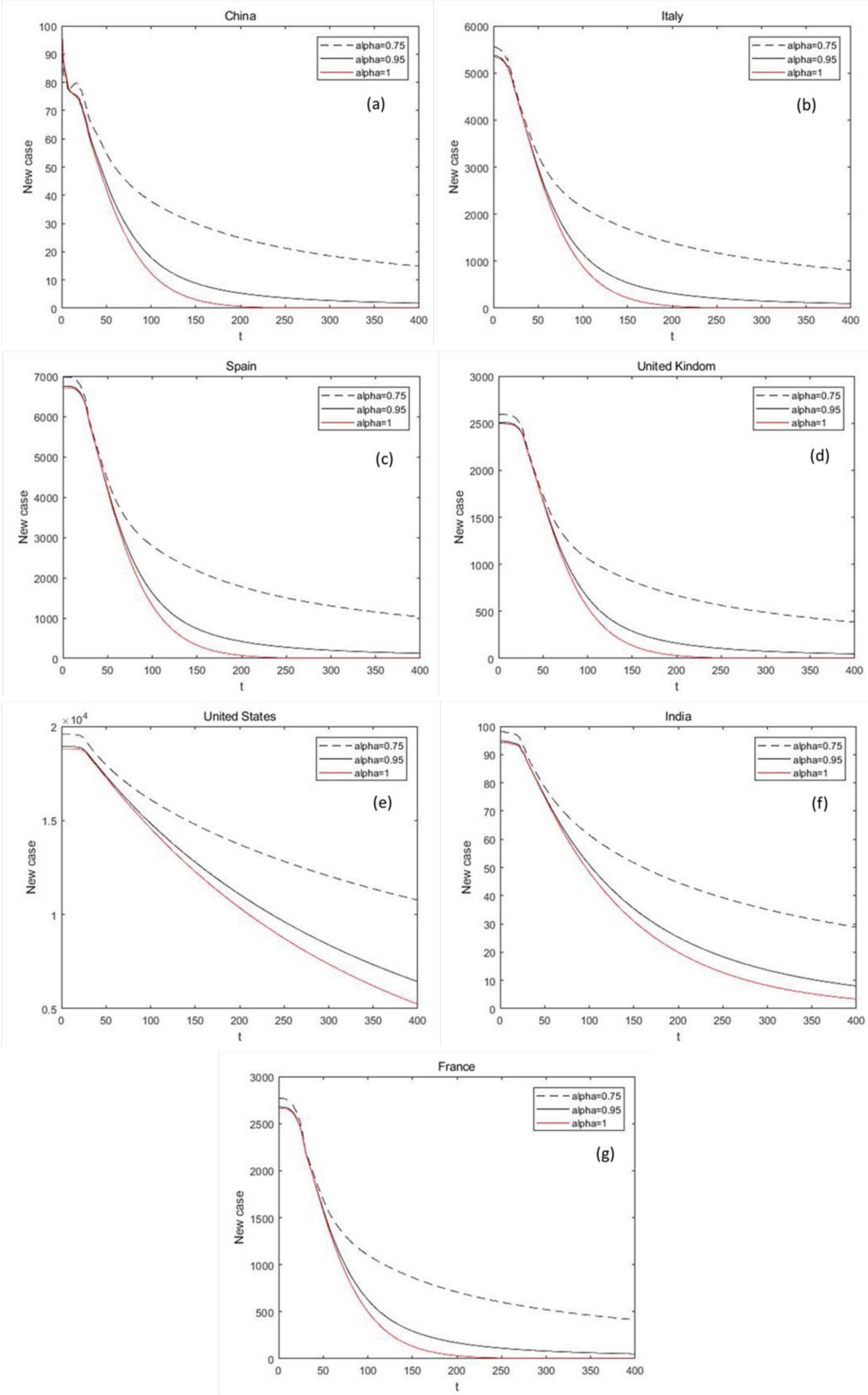

Fig. 4. New case forecast: China (a), Italy (b), Spain (c), United Kingdom (d), United States (e), India (f) and France (g).

4. Conclusion

As studied in Sardar et al.,[24] in the case of dengue transmission, as α approach 0, the time of disease elimination is longer. Similarly, in the prediction of COVID-19 by fractional order SIR model, the same trend is also shown. Our research shows that when predicting the trend of the number of infections, the closer α closes to 0, the longer it takes to eliminate the disease than α = 0:95 and α = 1. However, given a real dataset, how to determine the optimal alpha value is worth further discussion. Due to the uncertainty of parameter selection, the result may not be the best. Although all the parameters we selected meet the requirements in Syafruddin et al.,[25] in the future research, we still have to conduct a qualitative sensitivity analysis on these parameters to know how each parameter affects our disease changes. Because the fractional order has the nature of memory, in the simulation of COVID-19, fractional order is more practical than integer order.

Supporting Information

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Acknowledgements

The authors contributed equally to this study

Conflict of interests

The authors declare no conflict of interest.

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