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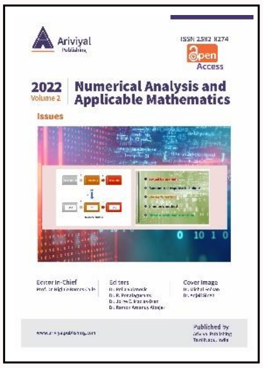

On Magneto-Rotatory Viscoelastic Convection

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Abstract: The aim of the present work was to study the effects of uniform vertical magnetic field and uniform rotation on the double-diffusive convection in Rivlin-Ericksen viscoelastic fluid through permeable media. Following the linearized stability theory, Boussinesq approximation and normal mode analysis, the dispersion relation is obtained. The stationary convection, stability of the system and oscillatory modes are discussed. For the case of stationary convection, it is found that the stable solute gradient and rotation have stabilizing effects on the system. In the presence of rotation, the medium permeability has a destabilizing (or stabilizing) effect and magnetic field has stabilizing (or destabilizing) effect on the system, whereas, in the absence of rotation, medium permeability and magnetic field have destabilizing effect and stabilizing effect on the system, respectively. The kinematic viscoelasticity has no effect for stationary convection. The kinematic viscoelasticity, rotation, stable solute gradient and magnetic field introduce oscillatory modes in the system, which were non-existent in their absence. The sufficient conditions for the non-existence of over stability are also obtained.

Keywords: Convection; Viscoelastic Fluid; Porous Medium; Uniform Magnetic Field; Uniform Rotation

1. Introduction

The theoretical and experimental results on thermal convection in a fluid layer, in the absence and presence of rotation and magnetic field have been given by Chandrasekhar.[1] Thermal convection is the most convective instability when crystals are produced from single element like silicon. However, gallium arsenide and other semi-conductors which require crystals made from compounds of elements are beginning to take on a prominent position in modern technologies. Hence, at present, there is a strong industrial demand for understanding the additional effects that can occur in the solidification of a mixture, which do not take place in one component system. The problem of thermohaline convection in a layer of fluid heated from below and subjected to a stable salinity gradient has been considered by Veronis.[2] The buoyancy force can arise not only from density differences due to variations in temperature but also from those due to variations in solute concentration. Double-diffusive convection problems arise in oceanography (salt fingers occur in the ocean when hot saline water overlies cooler fresher water which believed to play an important role in the mixing of properties in several regions of the ocean), limnology and engineering. The migration of moisture in fibrous insulation, bio/chemical contaminants transport in environment, underground disposal of nuclear wastes, magmas, groundwater, high quality crystal production and production of pure medication are some examples where double-diffusive convection is involved. Examples of particular interest are provided by ponds built to trap solar heat (Tabor and Matz[3]) and some Antarctic lakes (Shirtcliffe).[4] The physics is quite similar in the stellar case in that helium acts like salt in raising the density and in diffusing more slowly than heat. The conditions under which convective motions are important in stellar atmospheres are usually far removed from consideration of a single component fluid and rigid boundaries, and therefore it is desirable to consider a fluid acted on by a solute gradient and free boundaries.

The flow through porous media is of considerable interest for petroleum engineers, for geophysical fluid dynamicists and has importance in chemical technology and industry. An example in the geophysical context is the recovery of crude oil from the pores of reservoir rocks. Among the applications in engineering disciplines one can find the food processing industry, chemical processing industry, solidification and centrifugal casting of metals. Such flows has shown their great importance in petroleum engineering to study the movement of natural gas, oil and water through the oil reservoirs; in chemical engineering for filtration and purification processes and in the field of agriculture engineering to study the underground water resources, seepage of water in river beds. The problem of thermosolutal convection in fluids in a porous medium is of importance in geophysics, soil sciences, ground water hydrology and astrophysics. The study of thermosolutal convection in fluid saturated porous media has diverse practical applications, including that related to the materials processing technology, in particular, the melting and solidification of binary alloys. The development of geothermal power resources has increased general interest in the properties of convection in porous media. The scientific importance of the field has also increased because hydrothermal circulation is the dominant heat-transfer mechanism in young oceanic crust (Lister).[5] Generally it is accepted that comets consists of a dusty 'snowball' of a mixture of frozen gases which in the process of their journey changes from solid to gas and vice- versa. The physical properties of comets, meteorites and interplanetary dust strongly suggest the importance of porosity in the astrophysical context (McDonnel).[6]

The effect of a magnetic field on the stability of such a flow is of interest in geophysics, particularly in the study of Earth's core where the Earth's mantle, which consists of conducting fluid, behaves like a porous medium which can become convectively unstable as a result of differential diffusion. The other application of the results of flow through a porous medium in the presence of a magnetic field is in the study of the stability of a convective flow in the geothermal region. Also the magnetic field in double-diffusive convection has its importance in the fields of engineering, for example, MHD generators and astrophysics particularly in explaining the properties of large stars with a helium rich core. Stommel and Fedorov[7] and Linden[8] have remarked that the length scales characteristics of double-diffusive convective layers in the ocean may be sufficiently large that the Earth's rotation might be important in their formation. Moreover, the rotation of the Earth distorts the boundaries of a hexagonal convection cell in a fluid through a porous medium and the distortion plays an important role in the extraction of energy in the geothermal regions. Brakke[9] explained a double - diffusive instability that occurs when a solution of a slowly diffusing protein is layered over a denser solution of more rapidly diffusing sucrose. Nason et al.[10] found that this instability, which is deleterious to certain biochemical separations, can be suppressed by rotation in the ultracentrifuge. The fluid was considered to be Newtonian in the above studies.

Many common materials such as paints, polymers, plastics and more exotic one such as silicic magma, saturated soils and the Earth's lithosphere behaves as viscoelastic fluids. Due to the growing use of these viscoelastic materials in various manufacturing and processing industries, in geophysical fluid dynamics, in chemical technology and in petroleum industry, considerable effort has been directed towards understanding their flow. Oldroyd[11] proposed a theoretical model for a class of viscoelastic fluids. An experimental demonstration by Toms and Strawbridge[12] reveals that a dilute solution of methyl methacrylate in n-butyl acetate agrees well with the theoretical model of the Oldroyd fluid. Bhatia and Steiner[13] have studied the problem of thermal instability of Maxwellian viscoelastic fluid in the presence of rotation and have found that the rotation has a destabilizing influence in contrast to the stabilizing effect on an ordinary viscous (Newtonian) fluid. The thermal instability of an Oldroydian viscoelastic fluid acted on by a uniform rotation has been studied by Sharma.[14] Sharma and Sharma[15] studied the stability of the plane interface separating two Oldroydian viscoelastic superposed fluids of uniform densities. There are many elastico-viscous fluids that cannot be characterized by Oldroyd`s constitutive relations. Rivlin-Ericksen[16] fluids are such class of elastico-viscous fluids. It is well known that the Rivlin-Ericksen fluid[16] is characterized by the constitutive equations

$$S = -pI + \mu A_1 + \mu' A_2 + \mu^{ii} A_1^2 + \mu^{iii} A_2^2 + \mu^{iv}(A_1A_2 + A_2A_1) + \mu^{v}(A_1^2A_2 + A_2A_1^2) + \mu^{vi}(A_1A_2^2 + A_2^2 + A_1) + \mu^{vii}(A_1^2A_2^2 + A_2^2A_1^2) \tag{1}$$

where S is the Cauchy stress tensor, 'p' is an arbitrary hydrostatic pressure, I is the unit tensor and μ's are polynomial functions of the traces of the various tensors occurring in the representation, matrices 'A_1' and 'A_2' are defined by

$$[A_1]_{ij} = (q_{i,j} + q_{j,i}) \tag{2}$$

and

$$[A_2]_{ij} = \frac{\partial [A_1]_{ij}}{\partial t} + q_p[A_1]_{ij,p} + [A_1]_{ip}q_{p,j} + [A_1]_{pj}q_{p,i} \tag{3}$$

'q_p' being velocity vector.
On neglecting the squares and products of 'A_2', we have

$$S = -pI + \mu A_1 + \mu' A_2 + \mu^{ii} A_1^2, \tag{4}$$

where μ, μ^i and μ^{ii} are three material constants. It is customary to call μ the coefficient of ordinary viscosity, μ' the coefficient of viscoelasticity and μ^{ii}, the coefficient of cross-viscosity. The μ, μ^i and μ^{ii} are general functions of temperature and material properties. For many fluids such as aqueous solution of polycrylamid and poly-isobutylene, μ, μ^i and μ^{ii} may be taken as constants. Such and other polymers are used in the manufacture of parts of spacecrafts, aeroplane parts, tyres, belt conveyers, ropes, cushions, seats, foams, plastics, engineering equipment, adhesives, contact lens etc. Recently, polymers are also used in agriculture, communication appliances and in biomedical applications. Srivastava and Singh[17] have studied the unsteady flow of a dusty elastico-viscous Rivlin-Ericksen fluid through channels of different cross-sections in the presence of a time-dependent pressure gradient. In another study, Garg et al.[18] have studied the rectilinear oscillations of a sphere along its diameter in a conducting dusty Rivlin-Ericksen fluid in the presence of a uniform magnetic field. Sharma and Kumar[19] have studied the effect of rotation on thermal instability in Rivlin- Ericksen elastico-viscous fluid and found that rotation has a stabilizing effect and introduces oscillatory modes in the system.

Keeping in mind the importance in geophysics, soil sciences, ground water hydrology, astrophysics, chemical technology, industry and various applications mentioned above, the present paper, therefore, deals with the combined effect of uniform vertical magnetic field and uniform rotation on the double-diffusive instability of a Rivlin-Ericksen viscoelastic fluid in porous medium.

2. Structure of the Problem and Basic Equation

Consider an infinite, horizontal, incompressible Rivlin-Ericksen viscoelastic fluid layer of thickness, d, heated and soluted from below so that, the temperatures, densities and solute concentrations at the bottom surface $z=0$ are T_0, ρ_0 and C_0 and at the upper surface $z=d$ are T_d, ρ_d and C_d respectively, and that a uniform temperature gradient ($\beta=|dT/dz|$) and a uniform solute gradient ($\beta'=|dC/dz|$) are maintained. The gravity field $\vec{g}(0,0,-g)$, a uniform vertical magnetic field $\vec{H}(0,0,H)$ and a uniform vertical rotation $\vec{\Omega}(0,0,\Omega)$ pervade the system. This fluid layer is assumed to be flowing through an isotropic and homogeneous porous medium of porosity ε and medium permeability k_1.

Let $p,\rho,T,C,\alpha,\alpha',g,\eta,\mu_e$ and $\vec{q}(u,v,w)$ denote, respectively, the fluid pressure, density, temperature, solute concentration, and thermal coefficient of expansion, an analogous solvent coefficient of expansion, gravitational acceleration, resistivity, magnetic permeability and fluid velocity. The equations expressing the conservation of momentum, mass, temperature, solute concentration and equation of state of Rivlin-Ericksen viscoelastic fluid are

$$\frac{1}{\varepsilon}\left[\frac{\partial\vec{q}}{\partial t}+\frac{1}{\varepsilon}(\vec{q}.\nabla)\vec{q}\right]=-\left(\frac{1}{\rho_0}\right)\nabla\mathrm{p}+\vec{g}\left(1+\frac{\delta\rho}{\rho_0}\right)-\frac{1}{k_1}\left(v+v'\frac{\partial}{\partial t}\right)\vec{q}+\frac{\mu_e}{4\pi\rho_0}(\nabla\times\vec{H})\times\vec{H}+\frac{2}{\epsilon}(\vec{q}\times\vec{\Omega}), \tag{5}$$

$$\nabla.\vec{q}=0\,, \tag{6}$$

$$E\frac{\partial T}{\partial t}+(\vec{q}.\nabla)T=k\nabla^2T\ , \tag{7}$$

$$E'\frac{\partial C}{\partial t}+(\vec{q}.\nabla)C=k'\nabla^2C\ , \tag{8}$$

$$\rho=\rho_0[1-\alpha(T-T_0)+\alpha'(C-C_0)]\,, \tag{9}$$

where the suffix zero refers to values at the reference level $\mathrm{z}=0$ and in writing equation (5), use has been made of the Boussinesq approximation. The magnetic permeability μ_e, the kinematic viscosity v, the kinematic viscoelasticity v', the thermal diffusivity k and the solute diffusivity $\mathrm{k}'$are all assumed to be constants.

The Maxwell's equations yield

$$\varepsilon\frac{d\vec{H}}{dt}=(\vec{H}.\nabla)\vec{q}+\varepsilon\eta\nabla^2\vec{H}\ , \tag{10}$$

and

$$\nabla.\vec{H}=0\ \ , \tag{11}$$

where $\frac{d}{dt}=\frac{\partial}{\partial t}+\varepsilon^{-1}\vec{q}.\nabla$ stands for the Convective derivative.

Here $\mathrm{E}=\varepsilon+(1-\varepsilon)\left(\frac{\rho_s C_s}{\rho_0 C_f}\right)$ is a constant and E' is a constant analogous to E but corresponding to solute rather than heat. ρ_s, C_s and ρ_0, C_f stand for density and heat capacity of solid (porous matrix) material and fluid, respectively. The steady state solution is,

$$\vec{q}=(0,0,0)\ ,T=-\beta z+T_0,$$

$$C=-\beta' z+C_0\ ,\rho=\rho_0(1+\alpha\beta z-\alpha'\beta' z). \tag{12}$$

Consider a small perturbation on the steady state solution, and let $\delta p,\delta\rho,\theta,\gamma,\vec{h}\,(\mathrm{h_x},\mathrm{h_y},\mathrm{h_z})$ and $\vec{q}(u,v,w)$ denote, respectively, the perturbations in pressure p, density ρ, temperature T, solute concentration C, magnetic field $\vec{H}(0,0,H)$ and velocity $\vec{q}(0,0,0)$. The change in density $\delta\rho$, caused mainly by the perturbations θ and γ in temperature and concentration, is given by

$$\delta p=-\rho_0(\alpha\theta-\alpha'\gamma). \tag{13}$$

Then the linearized perturbation equations become

$$\frac{1}{\epsilon}\frac{\partial\vec{q}}{\partial t}=-\frac{1}{\rho_0}(\nabla\delta p)-\vec{g}(\alpha\theta-\alpha'\gamma)-\frac{1}{k_1}\left(\mathrm{v}+\mathrm{v}'\frac{\partial}{\partial t}\right)\vec{q}+\frac{\mu_e}{4\pi\rho_0}(\nabla\times\vec{h})\times\vec{H}\qquad+\frac{2}{\varepsilon}(\vec{q}\times\vec{\Omega}), \tag{14}$$

$$\nabla.\vec{q}=0\ , \tag{15}$$

$$E\frac{\partial\theta}{\partial t}=\beta w+k\nabla^2\theta\ , \tag{16}$$

$$E'\frac{\partial\gamma}{\partial t}=\beta' w+k'\nabla^2\gamma\ , \tag{17}$$

$$\varepsilon\frac{\partial\vec{h}}{\partial t}=(\vec{H}.\nabla)\vec{q}+\varepsilon\eta\nabla^2\vec{h}\ , \tag{18}$$

and

$$\nabla.\vec{h}=0\,. \tag{19}$$

3. The Dispersion Relation

For obtaining the dispersion relation, we now analyzing the disturbances into normal modes, assuming that the perturbation quantities are of the form

$$[w,h_z,\theta,\gamma,\zeta,\xi]=[W(z),K(z),\Theta(z),\Gamma(z),Z(z),X(z)]\exp(ik_x x+ik_y y+nt), \tag{20}$$

where k_x, k_y are the wave numbers along the $x-$ and $y-$ directions respectively, $k=\sqrt{(k_x^2+k_y^2)}$ is the resultant wave number and n is the growth rate which is, in general, a complex constant. $\zeta=\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}$ and $\xi=\frac{\partial h_y}{\partial x}-\frac{\partial h_x}{\partial y}$ stand for the z components of vorticity and current density, respectively.

Expressing the coordinates x, y, z in the new unit of length d and letting

$$a=kd,\sigma=\frac{nd^2}{\nu},p_1=\frac{\nu}{k},p_2=\frac{\nu}{\eta},q=\frac{\nu}{k'},F=\frac{\nu'}{d^2},P_l=\frac{k_1}{d^2}\ and\ D=\frac{d}{dz},$$

equations (14) – (19), using (20), became

$$\left[\frac{\sigma}{\varepsilon}+\frac{1}{P_l}(1+\sigma F)\right](D^2-a^2)W+\frac{ga^2d^2}{\nu}(\alpha\Theta-\alpha'\Gamma)-\frac{\mu_e Hd}{4\pi\rho_0\nu}(D^2-a^2)DK+\frac{2\Omega d^3}{\varepsilon\nu}DZ=0\,, \tag{21}$$

$$\left[\frac{\sigma}{\varepsilon}+\frac{1}{P_l}(1+\sigma F)\right]Z=\left(\frac{\mu_e Hd}{4\pi\rho_0\nu}\right)DX+\left(\frac{2\Omega d}{\varepsilon\nu}\right)DW\,, \tag{22}$$

$$(D^2-a^2-p_2\sigma)K=-\left(\frac{Hd}{\varepsilon\eta}\right)DW\,, \tag{23}$$

$$(D^2-a^2-p_2\sigma)X=-\left(\frac{Hd}{\varepsilon\eta}\right)DZ\,, \tag{24}$$

$$(D^2-a^2-Ep_1\sigma)\Theta=-\left(\frac{\beta d^2}{k}\right)W\,, \tag{25}$$

and

$$(D^2-a^2-E'q\sigma)\Gamma=-\left(\frac{\beta' d^2}{k'}\right)W. \tag{26}$$

Consider the case where both boundaries are free as well as perfect conductors of both heat and solute concentration, while the adjoining medium is perfectly conducting. The case of two free boundaries is a little artificial but it enables us to find analytical solutions and to make some qualitative conclusions. The appropriate boundary conditions, with respect to which equations (21) – (26) must be solved, are

$$W=D^2W=X=DZ=0,\Theta=0,\Gamma=0, at\ z=0\ and\ 1$$

$k=0$ on a perfectly conducting boundary

and h_x, h_y, h_z are continuous. (27)

The case of two free boundaries, though little artificial, is the most appropriate for stellar atmospheres.(Spiegel[20]) Using the above boundary conditions, it can be shown that all the even order derivatives of W must vanish for $z=0$ and 1 and hence the proper solution of W characterizing the lowest mode is

$$W = W_0 \sin \pi z\,, \tag{28}$$

where W_0 is a constant.

Eliminating $\Theta, \Gamma, \mathrm{K}, \mathrm{Z}$, and X between (21) – (26) substituting the proper solution (28) in the resultant equation, we obtain the dispersion relation

$$R_1 = \left(\frac{1+x}{x}\right)\left[\frac{i\sigma_1}{\varepsilon} + \frac{1}{P}(1 + i\sigma_1\pi^2 F)\right][1 + x + iEp_1\sigma_1] + Q_1\frac{(1+x)(1+x+iEp_1\sigma_1)}{x(1+x+ip_2\sigma_2)} +$$

$$S_1\frac{(1+x+iEp_1\sigma_1)}{(1+x+iEp\sigma_1)} + T_{A_1}\left[\frac{(1+x+iEp_1\sigma_1)(1+x+ip_2\sigma_1)}{x\left\{\left(\frac{i\sigma_1}{\varepsilon}+\frac{1}{P}(1+i\sigma_1\pi^2 F)\right)(1+x+ip_2\sigma_1)+Q_1\right\}}\right], \tag{29}$$

where

$$R_1 = \frac{g\alpha\beta d^4}{\nu k\pi^4}, S_1 = \frac{g\alpha'\beta' d^4}{\nu k'\pi^4}, Q_1 = \frac{\mu_e H^2 d^2}{4\pi\rho_0\nu\eta\varepsilon\pi^2}, T_{A_1} = \left(\frac{2\Omega d^2}{\varepsilon\nu\pi^2}\right), x = \frac{\alpha^2}{\pi^{2'}}$$

$$i\sigma_1 = \frac{\sigma}{\pi^2} \text{ and } P = \pi^2 P_l$$

Equation (29) is the required dispersion relation including the effects of magnetic field, rotation, medium permeability, kinematic viscoelasticity and stable solute gradient on the double-diffusive instability of Rivlin-Ericksen rotating viscoelastic fluid in porous medium in hydro magnetics.

4. Important Theorems and Discussion

Theorem 1: The system is stable or unstable.

Proof: Multiplying equation (21) by W^*, the complex conjugate of W and using equations (22) – (26) together with the boundary conditions (27), we obtain

$$\left[\frac{\sigma}{\varepsilon} + \frac{1}{P_l}(1+\sigma F)\right] I_1 + \left(\frac{g\alpha' k' a^2}{\nu\beta'}\right)[I_4 + E' q\sigma^* I_5] + \frac{\mu_e\varepsilon\eta}{4\pi\rho_0\nu}[I_6 + p_2\sigma^* I_7] + \frac{\mu_e\varepsilon\eta d^2}{4\pi\rho_0\nu}[I_8 + p_2\sigma I_9]$$
$$+ d^2\left[\frac{\sigma^*}{\varepsilon} + \frac{1}{P_l}(1+\sigma^* F)\right] I_{10} - \left(\frac{g\alpha k a^2}{\nu\beta}\right)[I_2 + Ep_1\sigma^* I_3] = 0\,, \tag{30}$$

where

$$I_1 = \int_0^1 (|DW|^2 + a^2|W|^2)dz, I_2 = \int_0^1 (|D\Theta|^2 + a^2|\Theta|^2)dz, I_3 = \int_0^1 (|\Theta|^2)dz\,,$$
$$I_4 = \int_0^1 (|D\Gamma|^2 + a^2|\Gamma|^2)dz,\ I_5 = \int_0^1 (|\Gamma|^2)dz,$$
$$I_6 = \int_0^1 (|D^2K|^2 + 2a^2|DK|^2 + a^4|K|^2)dz,\ I_7 = \int_0^1 (|DK|^2 + a^2|K|^2)dz,$$
$$I_8 = \int_0^1 (|DX|^2 + a^2|X|^2)dz,\ I_9 = \int_0^1 (|X|^2)dz,\ I_{10} = \int_0^1 (|Z|^2)dz. \tag{31}$$

The integrals $I_1, \ldots\ldots\ldots\ldots\ldots\ldots, I_{10}$ are all positive definite. Putting $\sigma = \sigma_r + i\sigma_i$ and equating the real and imaginary parts of equation (30), we obtain

$$\left[\left(\frac{1}{\varepsilon} + \frac{F}{P_l}\right) I_1 + \frac{g\alpha' k' a^2}{\nu\beta'} E' q I_5 + \frac{\mu_e\varepsilon\eta}{4\pi\rho_0\nu} p_2(I_7 + d^2 I_9) + d^2\left(\frac{1}{\varepsilon} + \frac{F}{P_l}\right) I_{10} - \frac{g\alpha k a^2}{\nu\beta} Ep_1 I_3\right]\sigma_r$$
$$= -\left[\frac{I_1}{P_l} + \frac{g\alpha' k' a^2}{\nu\beta'} I_4 + \frac{\mu_e\varepsilon\eta}{4\pi\rho_0\nu}(I_6 + d^2 I_8) + d^2\frac{1}{P_l} I_{10} - \frac{g\alpha k a^2}{\nu\beta} I_2\right], \tag{32}$$

$$\left[\left(\frac{1}{\varepsilon} + \frac{F}{P_l}\right) I_1 - \frac{g\alpha' k' a^2}{\nu\beta'} E' q I_5 - \frac{\mu_e\varepsilon\eta}{4\pi\rho_0\nu} p_2(I_7 - d^2 I_9) - d^2\left(\frac{1}{\varepsilon} + \frac{F}{P_l}\right) I_{10} + \frac{g\alpha k a^2}{\nu\beta} EP_1 I_3\right]\sigma_i = 0. \tag{33}$$

It is evident from equation (32) that σ_r is positive or negative. The system is, therefore, stable or unstable.

Theorem 2: The modes may be oscillatory or non-oscillatory in contrast to case of no magnetic field and rotation, and in the absence of kinematic viscoelasticity and stable solute gradient where modes are non-oscillatory.

Proof: Equation (33) yields that $\boldsymbol{\sigma_i}$ may be zero or non-zero, which means that the modes may be non-oscillatory or oscillatory. In the absence of kinematic viscoelasticity, stable solute gradient, rotation and magnetic field, equation (33) reduces to

$$\left[\frac{I_1}{\varepsilon}+\frac{g\alpha k a^2}{\nu\beta}Ep_1I_3\right]\sigma_i=0, \tag{34}$$

and the terms in brackets are positive definite. Thus $\sigma_i=0$, which means that oscillatory modes are not allowed and the principle of exchange of stabilities is satisfied for a porous medium, in the absence of kinematic viscoelasticity, stable solute gradient, rotation and magnetic field. This result is true for the porous as well as non-porous medium.(Chandrasekhar[1]) The oscillatory modes are introduced due to the presence of kinematic viscoelasticity, stable solute gradient, rotation and magnetic field, which were non-existent in their absence.

Theorem 3: The system is stable for $\frac{g\alpha k}{\nu\beta}\le\frac{4\pi^2}{P_l}$ and under the condition $\frac{g\alpha k}{\nu\beta}>\frac{4\pi^2}{P_l}$, the system becomes unstable.

Proof: From equation (33), it is clear that σ_i is zero when the quantity multiplying it is not zero and arbitrary when this quantity is zero.
If $\sigma_i\neq 0$, equation (32) upon utilizing (33) and the Rayleigh-Ritz inequality gives

$$\left[\frac{4\pi^2}{P_l}-\frac{g\alpha k}{\nu\beta}\right]\int_0^1|W|^2dz+\frac{\pi^2+a^2}{a^2}\left\{d^2\frac{1}{P_l}I_{10}+\frac{\mu_e\varepsilon\eta}{2\pi\rho_0\nu}p_2d^2I_9\sigma_r+\frac{\mu_e\varepsilon\eta}{4\pi\rho_0\nu}(I_6+d^2I_8)+\frac{g\alpha'k'a^2}{\nu\beta'}I_4+2\left(\frac{1}{\varepsilon}+\frac{F}{P_l}\right)\sigma_rI_1\right\}\le 0, \tag{35}$$

Since the minimum value of $\frac{(\pi^2+a^2)^2}{a^2}$ with respect to a^2 is $4\pi^2$.
Now, let $\sigma_r\ge 0$, we necessarily have from inequality (35) that

$$\frac{g\alpha k}{\nu\beta}>\frac{4\pi^2}{P_l}. \tag{36}$$

Hence, if

$$\frac{g\alpha k}{\nu\beta}\le\frac{4\pi^2}{P_l}, \tag{37}$$

then $\sigma_r<0$. Therefore, the system is stable.
Thus, under condition (37), the system is stable and under condition (36) the system becomes unstable.

Theorem 4: For stationary convection case:

- The stable solute gradient and rotation have stabilizing effects on the system.
- In the absence of rotation, the medium permeability has a destabilizing effect, whereas magnetic field has a stabilizing effect on the system.
- In the presence of rotation if

$$T_{A_1}<(or>)\frac{(1+x+PQ_1)^2}{(1+x)P^2}$$

the medium permeability has a destabilizing (or stabilizing) effect and magnetic field has a stabilizing (or destabilizing) effect on the system.

Proof: When the instability sets in as stationary convection, the marginal state will be characterized by $\sigma=0$. Putting $\sigma=0$, the dispersion relation (29) reduces to

$$R_1=\left(\frac{1+x}{x}\right)\left[\frac{1+x}{P}+Q_1\right]+T_{A_1}\frac{(1+x)^2}{x\left\{\frac{1+x}{P}+Q_1\right\}}+S_1, \tag{38}$$

which expresses the modified Rayleigh number R_1 as a function of the dimensionless wave x and the parameters S_1, T_{A_1}, Q_1 and P. The parameter F accounting for the kinematic viscoelasticity effect vanishes for the stationary convection.

To investigate the effects of stable solute gradient, rotation, magnetic field and medium permeability, we examine the behaviour of $\frac{dR_1}{dS_1},\frac{dR_1}{dT_{A_1}},\frac{dR_1}{dP}$ and $\frac{dR_1}{dQ_1}$ analytically.

(I) Equation (38) yields

$$\frac{dR_1}{dS_1}=+1, \tag{39}$$

which is positive. Thus, the stable solute gradient has a stabilizing effect on the system for the case of stationary convection.

Equation (38) also yields

$$\frac{dR_1}{dT_{A_1}} = \left(\frac{1+x}{x}\right)\frac{(1+x)}{\left\{\frac{1+x}{P}+Q_1\right\}}, \tag{40}$$

which is always positive. The rotation, therefore, has a stabilizing effect on the system for the case of stationary convection.

(II) It is evident from (38) that

$$\frac{dR_1}{dP} = \frac{(1+x)^2}{x}\left[\frac{1}{P^2} - T_{A_1}\frac{(1+x)}{(1+x+PQ_1)^2}\right], \tag{41}$$

and

$$\frac{dR_1}{dQ_1} = \frac{(1+x)}{x}\left[1 - T_{A_1}\frac{(1+x)P^2}{(1+x+PQ_1)^2}\right]. \tag{42}$$

In the absence of rotation $\left(\mathrm{T}_{\mathrm{A}_1} = 0\right)$, equation (41) reduces to

$$\frac{dR_1}{dP} = \frac{(1+x)^2}{xP^2}, \tag{43}$$

which is negative. Hence, the medium permeability has a destabilizing effect on the system in the absence of rotation. Now, in the absence of rotation, equation (42) gives

$$\frac{dR_1}{dQ_1} = \frac{(1+x)}{x}, \tag{44}$$

which is positive. The magnetic field, therefore, has a stabilizing effect on the system for the case of stationary convection in the absence of rotation.

(III) In the presence of rotation, it is clear from equations (41) and (42) that the medium permeability has a destabilizing (or stabilizing) effect and the magnetic field has a stabilizing (or destabilizing) effect on the system for the case of stationary convection if

$$T_{A_1} < (or >)\frac{(1+x+PQ_1)^2}{(1+x)P^2}. \tag{45}$$

Theorem 5: The sufficient conditions for the non-existence of over stability are

$$E\frac{v}{k} > min\left\{E'\frac{v}{k'}\frac{2}{\varepsilon}\left(\frac{2\Omega\pi k_1}{\varepsilon v d}\right)^2 (k_1+\varepsilon v')\right\}$$

and

$$v > \frac{k_{1\pi}}{\varepsilon^2 d^2}\left(\frac{3\mu_e H^2}{4v\rho_0}\right)(k_1+\varepsilon v).$$

Proof: Here we discuss the possibility of whether instability may occur as over stability. Since we wish to determine the critical Rayleigh number for the onset of instability via a state of pure oscillations, it suffices to find conditions for which (29) will admit of solutions with σ_1 real.

Equating the real and imaginary parts of equation (29) and eliminating R_1 between them, we obtain

$$A_4 c_1^4 + A_3 c_1^3 + A_2 c_1^2 + A_1 c_1 + A_0 = 0, \tag{46}$$

Where we have put $\mathrm{c}_1 = \sigma_1^2, \mathrm{b} = 1 + \mathrm{x}$ and

$$A_4 = E'^2 q^2 p_2^2\left(\frac{1}{\varepsilon}+\frac{\pi^2 F}{P}\right)^2\left[\left(\frac{1}{\varepsilon}+\frac{\pi^2 F}{P}\right)b + \frac{Ep_1}{P}\right], \tag{47}$$

$$A_3 = \left[\left(\frac{1}{\varepsilon}+\frac{\pi^2 E}{P}\right)^3 (2E'^2 q^2 p_2^2)\right]b^4 + \left[\frac{Ep_1 p_2^2}{P}\left(\frac{1}{\varepsilon}+\frac{\pi^2 F}{P}\right)^2 + 2E'^2 q^2\left\{\frac{Ep_1}{P}\left(\frac{1}{\varepsilon}+\frac{\pi^2 F}{P}\right)^2\right\}\right]b^3$$

$$+\left[E'^2 q^2 p_2\left(\frac{1}{\varepsilon}+\frac{\pi^2 F}{P}\right)\left\{\frac{p_2}{P_2} - 3Q_1\left(\frac{1}{\varepsilon}+\frac{\pi^2 F}{P}\right)\right\} + Ep_1 E'^2 q^2 Q_1\left(\frac{1}{\varepsilon}+\frac{\pi^2 F}{P}\right)^2\right]b^2$$

$$+p_2\left[E^2 q^2\left\{\frac{Ep_1}{2P}\left(\frac{p_2}{P_2} - 4Q_1\overline{\frac{1}{\varepsilon}+\frac{\pi^2 F}{P}}\right) + p_2\left(\frac{Ep_1}{2P^3} - T_{A_1}\overline{\frac{1}{\varepsilon}+\frac{\pi^2 F}{P}}\right)\right\} + S_1(b-1)p_2\left(\frac{1}{\varepsilon}+\frac{\pi^2 F}{P}\right)^2 (Ep_1 - E'q)\right]b$$

$$+\frac{T_{A_1}Ep_1E'^2q^2p_2^2}{P} \tag{48}$$

The coefficients $A_0 - A_2$ being quite lengthy and not needed in the discussion of over stability, have not been written here. Since σ_1 is real for overstability, the four values of $c_1(=\sigma_1^2)$ are positive. The sum of roots of equation (46) is $-\frac{A_3}{A_4}$, and if this is to be negative, then $A_3 > 0$ (since from (47), $A_4 > 0$).

It is clear from equation (48) that A_3 is always positive if

$$Ep_1 > E'q, Ep_1 > 2\mathrm{P}^3T_{A_1}\left(\frac{1}{\varepsilon}+\frac{\pi^2F}{P}\right) and\ p_2 > 3Q_1P^2\left(\frac{1}{\varepsilon}+\frac{\pi^2F}{P}\right), \tag{49}$$

which imply that

$$E\frac{v}{k} > E'\frac{v}{k'}, E\frac{v}{k} > \frac{2}{\varepsilon}\left(\frac{2\Omega\pi k_1}{\varepsilon vd}\right)^2(k_1+\varepsilon v')$$

and

$$v > \frac{k_1\pi}{\varepsilon^2\mathrm{d}^2}\left(\frac{3\mu_eH^2}{4v\rho_0}\right)(k_1+\varepsilon v'), \tag{50}$$

i.e.

$$E\frac{v}{k} > min\left\{E'\frac{v}{k'}\frac{2}{\varepsilon}\left(\frac{2\Omega\pi k_1}{\varepsilon vd}\right)^2(k_1+\varepsilon v')\right\}$$

and

$$v > \frac{k_1\pi}{\varepsilon^2\mathrm{d}^2}\left(\frac{3\mu_eH^2}{4v\rho_0}\right)(k_1+\varepsilon v'). \tag{51}$$

Thus

$$E\frac{v}{k} > min\left\{E'\frac{v}{k'}\frac{2}{\varepsilon}\left(\frac{2\Omega\pi k_1}{\varepsilon vd}\right)^2(k_1+\varepsilon v')\right\}$$

And

$$v > \frac{k_1\pi}{\varepsilon^2\mathrm{d}^2}\left(\frac{3\mu_eH^2}{4v\rho_0}\right)(k_1+\varepsilon v')$$

are the sufficient conditions for the non-existence of over stability, the violation of which does not necessarily imply the occurrence of over stability.

5. Conclusions

The double-diffusive instability of a Rivlin-Ericksen viscoelastic fluid in porous medium in the presence of uniform vertical magnetic field and uniform rotation is considered in the present paper. The investigation is motivated by its interesting complexities as a double- diffusion phenomena as well as its direct relevance to astrophysics and geophysics. The conditions under which convective motion in double-diffusive convection are important (e.g. in lower parts of the Earth's atmosphere, astrophysics, and several geophysical situations) are usually far removed from the consideration of a single component fluid and rigid boundaries and therefore it is desirable to consider a fluid acted on by a solute gradient and free boundaries. The main conclusions from the analysis of this paper are as follows:

- It is found that the kinematic viscoelasticity, rotation, stable solute gradient and magnetic field introduce oscillatory modes in the system which was non-existent in their absence.
- It is observed that the system is stable for $\frac{g\alpha k}{v\beta} \leq \frac{4\pi^2}{P_l}$ and under the condition $\frac{g\alpha k}{v\beta} > \frac{4\pi^2}{P_l}$, the system becomes unstable.
- For the case of stationary convection, the stable solute gradient and rotation are found to have stabilizing effects on the system.
- It is also observed for the case of stationary convection that in the presence of rotation, the magnetic field has a stabilizing (or destabilizing) effect and the medium permeability has a destabilizing (or stabilizing) effect under certain conditions whereas in the absence of rotation, the magnetic field and the medium permeability have stabilizing and destabilizing effects, respectively.
- The case of over stability is also considered. The conditions

$$E\frac{v}{k} > min\left\{E'\frac{v}{k'}\frac{2}{\varepsilon}\left(\frac{2\Omega\pi k_1}{\varepsilon vd}\right)^2(k_1+\varepsilon v')\right\}$$

and

$$v > \frac{k_1 \pi}{\varepsilon^2 d^2}\left(\frac{3\mu_e H^2}{4 v \rho_0}\right)(k_1 + \varepsilon v'),$$

are the sufficient conditions for the non-existence of over stability, the violation of which does not necessarily imply the occurrence of over stability.

Conflicts of Interest

The authors declare no conflict of interest.

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