Nano Progress

Research Article

DOI: 10.36686/Ariviyal.NP.2021.03.06.030

Nano Prog., (2021) 3(6), 19-34.

A Few Notes on the Wave Absorption Applied to Cloaking Problem

Vladimir V. Arabadzhi

Institute of Applied Physics (RAS), 603950, Box-120, Nizhny Novgorod, Ulianov st. 46, Russian Federation.

***Corresponding author E-mail address**: v.v.arabadzhi@appl.sci-nnov.ru

ISSN: 2582-1598

Crossref

Publication details
Received: 19th May 2021
Revised: 29th July 2021
Accepted: 29th July 2021
Published: 16th August 2021

Abstract: J. Pendry's perfect cloaking solution is briefly considered. Other approaches with absorption of waves are considered too: absorbing coatings, black holes,… In addition, well-known approaches to describing black body diffraction are discussed for sound waves and electromagnetic waves as waves without dispersion. The formulations of black body definition of are corrected as the review progresses. The alternative conception of "black body" (in the wave diffraction sense) is represented in this article for electromagnetic waves. For several decades, many researchers have tried to find a structure (constant in time, and with field representation by complex amplitudes at any frequency) of an absorbing shell that would satisfy simultaneously (jointly) the following conditions: (a) effective absorption (and cloaking); (b) a spatial ultra-wide absorption band (i.e. the absorption efficiency is independent of the spatial frequency or of incident wave direction), (c) an ultra-wide absorption frequency band (i.e. the absorption efficiency does not depend on the incident wave time frequency), (d) the small thickness of the absorbing coating compared to the length of the absorbed wave and to the geometric dimension of protected body. But without full success (i.e. without (a), (b), (c), (d) all together), because: (1) any wave to be absorbed need time (more or equal to its period) and distance (more or equal to its wavelength) to have time to make a work (if we do not make conversion its frequency) on the absorber; (2) in passive systems with parameters constant in time, the interaction of absorbing elements at the frequency of the incident wave is inevitable. Now remind that in all these years microelectronic technologies (designed for computers and according Gordon Moore's law) have been intensively developed: the miniature and rate of the element base (or the spatial-temporal resolution). On the other hand wavelengths that were intended to be absorbed by the "black" shells remained the same due to the constant conditions of the long-range propagation of these waves. This work is an attempt to use the great successes of nano-electronics to satisfy conditions (a)-(d) jointly. The required level of nano-electronics development is very high, but quite real today.

Keywords: simultaneous; dissection; instantaneous; spatial, distribution; contour; switched; boundary

1. Introduction

Cloaking generally refers to certain actions and devices designed to make it difficult to detect a physical body or make it less visible against some background, such as camouflage in soldiers or coloration in animals. This article will consider the following formulation of the cloaking problem for linear non-dispersive waves (acoustic or electromagnetic for instance, the choice is motivated by the simplicity of presentation). A plane wave falls on the protected body $\hat{D}$ (with a characteristic linear size $\sim D$) in the direction $\mathbf{w}(|\mathbf{W}|=1)$ (the field of this wave in the absence of the body $\hat{D}$ is equal to $U_w(\omega,\mathbf{r})$, Fig. 1-a) with the frequency and wavelength $\lambda = 2\pi c/\omega$ (c -speed of wave propagation). Below we call cloaking various ways to reduce the distortion (scattering) of the incident wave field U_w by the body in the far zone of the body $\hat{D}$, i.e. at a distance $r \gg D^2/\lambda(r-|\mathbf{r}|)$ from the body $\hat{D}$. In this article, we will consider various methods of masking using a masking layer $\hat{L}$ (with an outer surface $\hat{\mathrm{S}}_{\mathrm{B}}$, an inner surface $\hat{\mathrm{S}}_{\mathrm{L}}$ and a minimum thickness

$$L = \min_{\bar{r}\in\hat{\mathrm{S}}_{\mathrm{B}},\bar{\bar{r}}\in\hat{\mathrm{S}}_{\mathrm{L}}} |\bar{r}-\bar{\bar{r}}|$$

or a cloaking box $\hat{\mathrm{L}}$ (below we also will call this as "layer", "shell", "area" too) in which we must place the protected body $\hat{D}$. From outside the cloaking box $\hat{\mathrm{L}}$ occupies an area of space $\hat{\mathrm{B}}$ bounded by the outer surface $\hat{\mathrm{S}}_{\mathrm{B}}$ (with an outer linear dimension B); the area $\hat{\mathrm{B}}\backslash\hat{\mathrm{L}}$ is intended to place the protected body $\hat{D}$. The masking box $\hat{\mathrm{L}}$ (layer $\hat{\mathrm{L}}$) must simultaneously satisfy two conditions: be invisible from the outside (i.e. the outer far scattering field $U_s = U - U_{\mathrm{w}}$ (to be reduced) of box $\hat{\mathrm{L}}$, propagating radially in all directions $\mathbf{s}(|\mathbf{s}|=1)$); be opaque for external waves (the wave field $U(\mathbf{r}\,\epsilon\,\hat{\mathrm{B}}\backslash\hat{\mathrm{L}}=0)$) for any $U_{\mathrm{w}} \neq 0$,or:

$$\left((U)_{\mathbf{r}\notin\hat{\mathrm{B}}} = U_{\mathrm{w}}\right) \cap \left((U)_{r\in\hat{\mathrm{B}}\backslash\hat{\mathrm{L}}} = 0\right). \quad (1)$$

The zero fields inside the cloaking box is needed in order to be placed in $\hat{\mathrm{B}}\backslash\hat{\mathrm{L}}$ the protected body $\hat{D}$ with arbitrary scattering

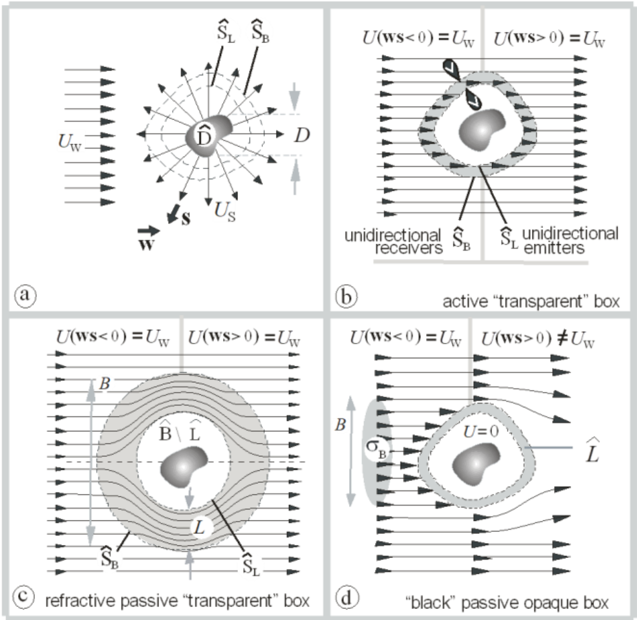

Fig. 1. (a) Power flux in a plane incident wave U_w (with direction $\mathbf{w}$) and in the far scattering field U_s (in radial directions $\mathbf{s}$) due to a body $\hat{D}$ with a characteristic size ~D; (b) active suppression of the scattering field using unidirectional emitters and unidirectional receivers; (c) passive stray field based on the negative refractive index of the shell $\hat{L}$; (d) suppression of the backscattering field using an absorbing shell $\hat{L}$.

characteristics. We assume that in order to satisfy conditions (1), we can create a special filling inside the layer $\hat{L}$ in the form of a special distribution $\hat{\Xi}$ of the parameters of the layer $\hat{L}$: in Sections 1 to 3, this is a time-constant spatial distribution $\hat{\Xi}(\mathbf{r})$ of parameters, and in **Section 4**, this is a time-variable spatial distribution $\hat{\Xi}(\mathbf{r},t)$ distribution of layer parameters. Generally speaking, as a result of minimizing scattering, the distribution can also depend on the direction $\mathbf{w}$ and frequency ω of the incident wave, i.e. $\hat{\Xi}(\mathbf{r},\mathbf{w},\omega)$. The quality of cloaking will be further evaluated by either the value $\bar{\Re} = \sigma_S/\sigma_B$ (or the value $\Re = \sigma(\mathbf{ws} < \mathbf{0})/\sigma_B$) that needs to be minimized somehow, i.e. $\bar{\Re} \to 0$ or either $\Re \to 0$. Here $\sigma_S = \sigma(\mathbf{ws} < \mathbf{0}) + \boldsymbol{\sigma}(\mathbf{ws} > \mathbf{0})$ is the area of the scattering cross-section of the cloaking box $\hat{L}$, $\sigma_B \sim B^2$ is the cross-section of the box $\hat{L}$ projected onto the plane of the front of the incident wave U_w (in the general case $\sigma_B = \sigma_B(\mathbf{w})$), $\sigma(\mathbf{ws} < 0) = W_S(\mathbf{ws} < 0)/W_w$ is the area of the backscattering cross-section of the box $\hat{L}$, $W_S(\mathbf{ws} < 0)$ is the power of the scattering field going into the rear infinite half-space, W_w is the power flux density in the plane incident wave U_w, $\sigma(\mathbf{ws} > 0) = W_s(\mathbf{ws} > 0)/W_w$ is the cross-sectional area of the forward scattering of the box $\hat{L}$, $W_s(\mathbf{ws} > 0)$ is the power of the scattering field going into the forward infinite half-space.

The purpose of this article is to find opportunities for creating such a cloaking box $\hat{L}$ that can simultaneously satisfy (i.e. $(1C) \cap (2C) \cap (3C) \cap (4C) \cap (5C)$) the following conditions:

(1C) $\bar{\Re} = \sigma_S/\sigma_B \ll 1$ (or $\Re = \sigma(\mathbf{ws} < 0)/\sigma_B \ll 1$);
(2C) Fulfillment of requirement (1C) within range $\hat{\omega}$ of frequencies ω should not require restructuring of the functional structure $\hat{\Xi}$ of the layer $\hat{L}$ for a given frequency ω, i.e. $\hat{\Xi} = \hat{\Xi}(\mathbf{r}) \neq \hat{\Xi}(\mathbf{r},\omega)$. The frequency range $\omega \epsilon \hat{\omega}[\omega_{min} \leq \omega \leq \omega_{max}]$ and wavelength $\lambda = 2\pi c/\omega$ range $\lambda \epsilon \hat{\lambda}[\lambda_{min} \leq \lambda \leq \lambda_{max}$ must be so wide that the conditions $B / \lambda_{min} \gg 1$and $\lambda_{max}/B \gg 1$ are met, where ω_{min}, ω_{max}, $\lambda_{min} = 2\pi c/\omega_{max}$, $\lambda_{max} = 2\pi c/\omega_{min}$ are the bounds of above ranges;
(3C) Fulfillment of requirement (1C) within a range $\mathbf{w}\epsilon\hat{\mathbf{W}}$ of directions $\mathbf{w}$ should not require restructuring of the functional structure $\hat{\Xi}$ of the layer $\hat{L}$ for a given direction $\mathbf{w}$, i.e. $\hat{\Xi} = \hat{\Xi}(\mathbf{r}) \neq \hat{\Xi}(\mathbf{r},\omega)$;
(4C) The surface $\hat{S}_B$ has any shape and size;
(5C) The minimum wall thickness L of the box $\hat{L}$ is small compared to the minimum wavelength of the range $\lambda\epsilon\hat{\Lambda}$, i. e. $L \ll \lambda_{min}$.

In other words, conditions (2C), (3C), (4C) assume the functional homogeneity of the layer $\hat{L}$ filling $\hat{\Xi} = \hat{\Xi}(\mathbf{r})$in the tangential direction (with respect to the surface $\hat{S}_B$).

2. Three known ideas for scattering suppression

In **Fig. 1**, three possible options for cloaking boxes are presented: an active cloaking box ($\bar{\Re} \to 0$, by Mangiante [1], **Fig. 1-b**), a passive refraction cloaking box ($\bar{\Re} \to 0$, by Pendry [2], **Fig. 1-c**), a passive absorbing cloaking box ($\bar{\Re} \to 0$, by Kirchhoff [3], **Fig. 1-d**), where "passive" means the zero right part in wave equations and boundary conditions, "active" means the corresponding nonzero right part.

(1) There is an active (this means source control on the right-hand side of the wave equation or boundary conditions) solution (**Fig. 1-b**) to the problem of acoustic masking, investigated by Mangiante G.[1] for spatially one dimensional case: a specially selected combination of one-sided emitting sources (and one-sided receivers) creates a field $U = U_w$ outside the box $\hat{B}$ and a full (together with incident wave) field $U = 0$ inside the box $\hat{B}$ (i.e. $\mathbf{r} \, \epsilon \, \hat{B} \setminus \hat{L}$). Unidirectional emitters and receivers ensure system stability. The main disadvantage of this approach is that there is no explicit causal analytical expression for the complex amplitudes of the sources (on the frequency ω) in terms of the measured field values for 2-3 dimensional cases, where for this it is necessary to solve the integral equations over the surfaces $\hat{S}_B$ and $\hat{S}_L$ the integral equation for the complex amplitudes of the emitters and the signals of the receivers.

(2) In the passive (right parts of wave equation and boundary conditions are zero) solution obtained by Pendry et al. (see section 2 below and Fig. 1-c)), the layer $\hat{L}$ forces the incident wave (rays) to flow around the region $\hat{B} \setminus \hat{L}$ without distorting the field from the outside $\hat{B}$.

(3) Kirchhoff black body approximation: backscatter suppression based on the absorption of the incident wave energy (see section 3 and Fig. 1-d).

3. On the perfect solution to the masking problem

Speaking of masking, it is impossible not to dwell briefly on a very beautiful solution (for sound and for electromagnetic waves), described, for example, in,[2] although there is no absorption of waves or any dissipative elements at all. In this work, were found

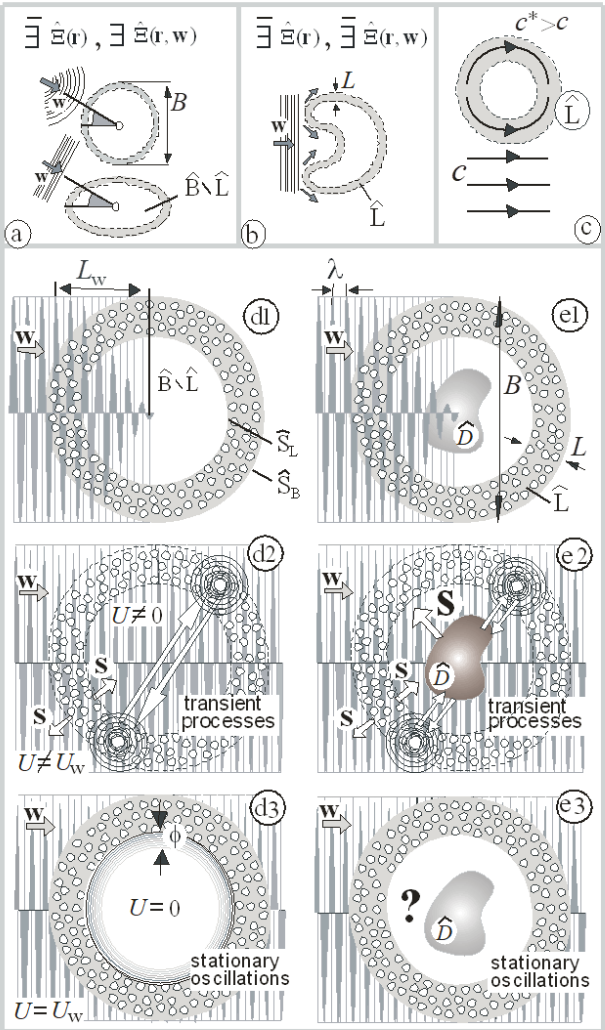

Fig. 2. To the different cases of perfect cloak: (a) a non-spherical shell or a non-planar incident wave lead to a dependence of the shell structure $\hat{\Xi}(\mathbf{r},\mathbf{w})$ on the direction $\mathbf{w}$ of incident wave; (b) concave shell shape is unacceptable; (c) anisotropic difference in phase velocities (c and $c^* = c\pi/2$) on different trajectories causes inevitable wave dispersion $k = k(\omega)$. Transient modes of with empty area $\hat{B}\backslash\hat{L}$ (d): (d1) $0 < t < B/c$, incident wave is entering the shell; (d2) $t > B/c$, establishing interaction between shell elements, scattering is not limited; (d3) $t \gg B/c$, the desired stationary mode of oscillations. Transient modes with body $\hat{D}$ to be protected inside $\hat{B}\backslash\hat{L}$ (e): (e1) entering the shell; (e2) establishing interaction between shell elements, scattering is not limited; (e3) stationary mode of oscillations (desired mode?).

some special distribution (for the frequency ω of *plane* incident wave), constant in time spherically symmetric distribution $\hat{\Xi} = \hat{\Xi}(\mathbf{r},\omega)$, of parameters inside spherical layer $\hat{L}$ (or cloaking shell $\hat{L}$) with negative refractive index. Due to this distribution $\hat{\Xi}(\mathbf{r},\omega)$ of parameters, a plane incident wave (falling on a layer $\hat{L}$) is carried away into the layer $\hat{L}$ without penetrating into the inner spherical (or cylindrical) region $\hat{B}\backslash\hat{L}$ of the layer $\hat{L}$ and goes outside in the form of the same plane wave (Fig. 1-c). The incident wave, as it were, flows around the region $\hat{B}\backslash\hat{L}$ $(U)_{\mathbf{r}\epsilon\hat{B}\backslash\hat{L}} = 0$, or more precisely speaking $|(U)_{\mathbf{r}\epsilon\hat{B}\backslash\hat{L}}| \ll |U_{\mathrm{w}}|$) penetrating only into the shell $\hat{L}$, and leaves it without any distortion (without scattering, i.e. $(U)_{\mathbf{r}\epsilon\hat{B}} = U_{\mathrm{w}}$ or more precisely speaking $|(U)_{\mathbf{r}\epsilon\hat{B}} - U_{\mathrm{w}}| \ll |U_{\mathrm{w}}|$). This is an ideal cloaking for all directions $\mathbf{w}$ of incident wave and one frequency ω. The combination of a reflectorless entry ($(U)_{\mathbf{r}\epsilon\hat{B}} = U_{\mathrm{w}}$) of waves into the shell $\hat{L}$ with an appointed channelization ($(U)_{\mathbf{r}\epsilon\hat{B}\backslash\hat{L}} = 0$) and refraction is possible only at one frequency ω for special structure $\hat{\Xi}(\mathbf{r},\omega)$, another frequency ω requires another structure $\hat{\Xi}(\mathbf{r},\omega)$. This is due to inevitable (for the systems with *parameters constant in time*) *interaction of shell elements on the frequency of incident wave*. But the fulfillment of above two conditions even at least at one frequency ω (for the structure $\hat{\Xi}(\mathbf{r},\omega)$) is a remarkable fact!

Below we will present several limitations on applications of this solution. Perfect cloaking[2] forces the power flux lines of the incident wave from the cross-sectional area $B^2\pi/4$ of the ball of radius $B/2$ to concentrate in a flat ring with radii $B/2$ and $(B/2) - L$ (see Fig. 1-c). Therefore, at $L \ll B$ (as we want, see Fig. 1-c) the intensities of the wave fields inside the cloaking shell become in $\pi B/4L \gg 1$ times greater than in the incident wave, and can go beyond the limitations of the linearity of the medium.

The non-sphericity of the shell $\hat{L}$ surface $\hat{S}_{\mathrm{B}}$ (Fig. 2-a, even with a plane front of the incident wave), as well as the non-sphericity of the incident wave (Fig. 2-b, even with the sphericity of the shell $\hat{L}$) cause an undesirable dependence of the shell $\hat{L}$ structure $\hat{\Xi}(\mathbf{r},\omega,\mathbf{W})$ on the direction $\mathbf{w}$ of the incident wave (∃ means “exist”, ∄ means “does not exist”). Concave surface $\hat{S}_{\mathrm{B}}$ areas (Fig. 2-d) completely contradict the solution.[2] Note that the difference in wave velocities inside the shell (for example, c and $c *= c\pi/2 > c$ as shown in Fig. 2-c) means anisotropy, and anisotropy inevitably gives rise to wave dispersion and a phase decay in structure $\hat{\Xi}(\mathbf{r},\omega_0)$ efficiency, for frequencies $\omega \neq \omega_0$.

Until now, we have assumed that the incident wave has always been (monochromatic problem). Now we will try to consider the problem with initial conditions and assume that the front (width $L_{\mathrm{w}} > \lambda$) of a plane incident wave U_{w} with the frequency ω and wavelength λ moves towards the shell $\hat{L}$ in the direction $\mathbf{w}$, and, in one case (Fig. 2-d) before contact with the incident wave, the area $\hat{B}\backslash\hat{L}$ is empty (i.e., filled with an external homogeneous medium), and in another case (Fig. 2-e), the protected body $\hat{D}$ (scattering body $\hat{D}$) was already in the area $\hat{B}\backslash\hat{L}$ before contact with the incident wave. So we will assume that at the moment $t = 0$ the leading front of the incident wave have touched by its front point to the nearest point of the surface $\hat{S}_{\mathrm{B}}$.

Limitation on the wave thinness L/λ of the cloak $\hat{L}$: generally speaking, we can consider the monochromatic masking problem (fulfilling the condition $(U)_{\mathbf{r}\notin\hat{B}} = U_{\mathrm{w}})\cap((U)_{\mathbf{r}\epsilon\hat{B}\backslash\hat{L}} = 0)$ as an analog of antennas synthesis in the case of superdirectivity).[4] Hypersensitivity to the accuracy of manufacturing oscillators can occur with a small wave thickness ($L/\lambda \ll 1$) of the shell $\hat{L}$. A stable synthesis (admissible sensitive to implementation errors, very finest tuning) of such a distribution is possible with a sufficiently large wave thickness of the shell ($L/\lambda \ll 1$), i.e. in the ray approximation.

Now to begin with, we note that a metamaterial (shell $\hat{L}$ structure $\hat{\Xi}$) can be considered as a big set of discrete oscillators (as usually happens in practice) located at separate points from each other and interconnected between each other by both wave fields and special dynamic circuits (independent of time). The incident wave generates oscillations of these oscillators (or shell $\hat{L}$ elements).

Solution[2] becomes relevant (i.e.,$\{(U)_{\mathbf{r}\notin\hat{\mathrm{B}}}=U_{\mathrm{w}}\}\cap\{(U)_{\mathbf{r}\in\hat{\mathrm{B}}\setminus\hat{\mathrm{L}}}=0\}$) only when (at $t\gg B/c$ atleast) the vibrations of the shell $\hat{\mathrm{L}}$ elements have reached their stationary complex amplitudes, and *in the absence of any scattering (opaque) body* $\hat{\mathrm{D}}$ *in the region* $\hat{\mathrm{B}}\setminus\hat{\mathrm{L}}$.

In Fig. 2-d and Fig.2-e schematically represented three states of the system (shell $\hat{\mathrm{L}}$): $0<t<B/c$ onset, $0<t<B/c$ tuning through the interaction of oscillators (transient process), $t\gg B/c$ stationary (monochromatic at the frequency of the incident wave) oscillations of all elements of the shell. An important feature of the monochromatic (stationary) problem is the fact that each oscillator already “knows” about the oscillations of each other oscillator (unlike the transient process), so we need $L_{\mathrm{w}}\gg B$. Comparing Fig. 2-d and Fig. 2-e, it is easy to see that: if the tuning of the oscillators (shell $\hat{\mathrm{L}}$ in the absence of a body $\hat{\mathrm{D}}$) to a stationary mode (in which the condition $\{(U)_{\mathbf{r}\notin\hat{\mathrm{B}}}=U_{\mathrm{w}}\}\cap\{(U)_{\mathbf{r}\in\hat{\mathrm{B}}\setminus\hat{\mathrm{L}}}=0\}$ is satisfied due to the interference of fields of all oscillators) occurs in time $t\gg B/c$ (this means the frequency band width $\Delta\omega\ll 2\pi c/B$). Then in the presence of a scattering body $\hat{\mathrm{D}}$, the stationary amplitudes and phases of oscillations of the oscillators are unlikely to coincide with similar values in the absence of a masked body $\hat{\mathrm{D}}$. Dissipation in the solution[2] is absent, but the damping of transient processes occurs due to the radiation of oscillators to infinity.

On the other hand, the reasoning just above presented should not be taken into account if the wave thickness $L/\lambda\gg 1$ of the shell $\hat{\mathrm{L}}$ is large and the layer $\hat{\emptyset}$ (area of thickness $\emptyset$, see Fig. 2-d3) of the reactive field of the oscillators in the region does not touch the body $\hat{\mathrm{D}}$. However, as mentioned in Section 1, we are interested in the possibility of effective cloaking at $L/\lambda\ll 1$. Thus, the solution discussed above satisfies condition (1C), but cannot simultaneously satisfy conditions (1C)-(5C) formulated in Section 1, due to the inevitable (for a systems with constant parameters in time) interaction of the shell $\hat{\mathrm{L}}$ elements at the frequency of the incident wave.

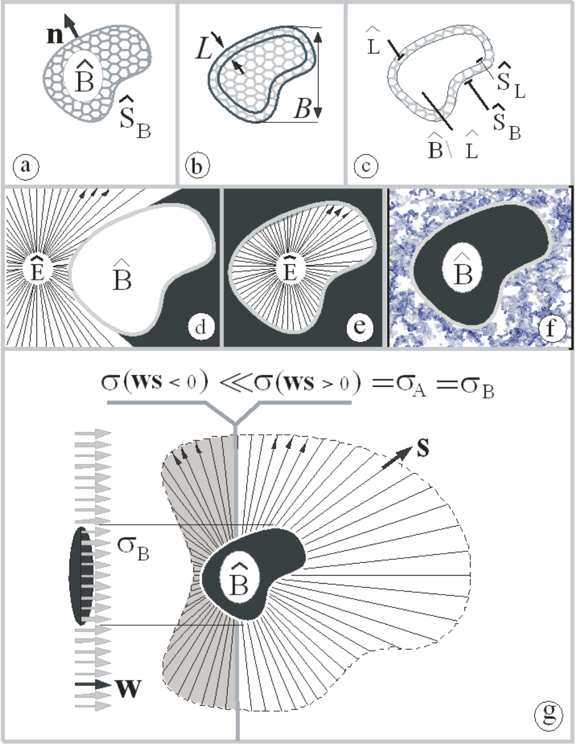

Fig. 3. To the Kirchhoff's definition of a black body: (a) a black body $\hat{\mathrm{B}}$ with a surface $\hat{\mathrm{S}}_{\mathrm{B}}$; (b) an "infinitely thin" absorbent (black) layer $\hat{\mathrm{L}}$ of thickness L; (c) masking black box, a cavity $\hat{\mathrm{B}}\setminus\hat{\mathrm{L}}$ with zero field inside the inner surface $\hat{\mathrm{S}}_{\mathrm{L}}$ of the black layer $\hat{\mathrm{L}}$; (d) an observer (or source $\hat{\mathrm{E}}$) outside the black body, and in the region $\hat{\mathrm{B}}\setminus\hat{\mathrm{L}}$ there is zero field; (e) an observer (or source $\hat{\mathrm{E}}$) inside the area $\hat{\mathrm{B}}\setminus\hat{\mathrm{L}}$ "sees" free space as in anechoic chamber ; (f) no shadow in the diffuse incident wave field; (g) the case of a plane incident wave with direction $\mathbf{w}$ and frequency ω, $\sigma_B(\mathbf{w})$ is the geometric cross section of the black body, σ_A is the absorption cross section of the black body, $\sigma(\mathbf{ws}<0)$ is the cross section for scattering into half-space $\mathbf{ws}<0$, and $\sigma(\mathbf{ws}>0)$ is the cross section for scattering into half-space $\mathbf{ws}>0$.

4. Review of the Concepts of a Black Body with Parameters Constant in Time

Below we will consider various approaches to the cloaking problem based on the absorption (unlike Section 2) of incident waves. Absorption is the transformation of the energy of the incident waves into energy of a different nature, for example, into heat. If the incident wave performs work on any device, then its energy is converted into heat and energy of scattered wave field. We begin with the concept of a black body (BB), which plays a fundamental role in the problem of cloaking.

4.1. Definition of Black Body

Kirchhoff in 1860 introduced the theoretical concept of a perfect black body with a completely absorbing surface layer of infinitely small thickness, but Planck noted some severe restrictions upon this idea.[3,5,6] What physical body is capable of satisfying these conditions formulated by Kirchhoff? The search for an answer to this question goes in two directions (generally speaking connected with each other) of research: ($\mathbf{r}\in\hat{\mathrm{B}}$) ***the internal problem of black body*** concerns the searching of BB’s internal functional structure $\hat{\Xi}(\mathbf{r}\in\hat{\mathrm{L}})$ (the process of converting wave energy into heat in the layer $\hat{\mathrm{L}}$); ($\mathbf{r}\in\hat{\mathrm{L}}$) ***external problem of black body*** concerns the diffraction of incident waves on the BB (traditionally this is the formulation of boundary conditions on BB’s surface $\hat{\mathrm{S}}_{\mathrm{B}}$).

4.2. Black Shell

We note immediately that, according to Kirchhoff's definition, all the fundamental properties and functions of a black body $\hat{\mathrm{B}}$ are concentrated in a thin absorbing layer $\hat{\mathrm{L}}$ (black body (Fig. 3-a) → black shell (Fig. 3-b) → black box (Fig. 3-c)), and thickness L is considered small relating to all scales including to the length λ of the incident wave (recall that this condition coincides with the cloaking requirement (1C)-(5C) in above section 1). So a black shell $\hat{\mathrm{L}}$ is a full equivalent of black body (Fig. 3-d) in relation to an outside observer at a point $\mathbf{r}\notin\hat{B}$, and the black shell $\hat{\mathrm{L}}$ represents a free space (Fig. 3-e) or an anechoic chamber in relation to the inner observer at the point $\mathbf{r}\in[\hat{\mathrm{B}}\setminus\hat{\mathrm{L}}]$. Figs. 3-d, 3-e presents the cases of spatially coherent incident waves (spherical). Fig. 3-f present the case of spatially incoherent (or diffuse one) incident wave, where the shadow does not arise at all. On the other hand the field in the region $\hat{\mathrm{B}}\backslash\hat{\mathrm{L}}$ is always zero $U = 0$ (i.e., a masked body can be placed there). Unlike the ideal masking described in Section 2., the zero field $U = 0$ in the protection area $\hat{\mathrm{B}}\backslash\hat{\mathrm{L}}$ is created not by superposition of the scattering fields of individual elements (oscillators) of the shell $\hat{\mathrm{L}}$, but by absorption in it (conversion into heat). Max Planck[6] noticed, that Kirchhoff's perfect black bodies that absorb all the radiation that falls on them cannot be realized in an infinitely thin surface layer, and impose conditions upon scattering of the light within the black body that are difficult to satisfy.

We would dare to add a few simple words to this note by Max Planck: (1) any wave (of length λ) to be absorbed (to make work against dissipative forces and be converted into heat) needs spatial interval about $\geq \lambda$ (and time interval about $\geq \lambda/c$). Determination of BB according to Kirchhoff works well at large wave sizes of the body ($B \gg \lambda_{max}$) and large wave thickness of the absorbing layer ($L \gg \lambda_{max}$). However, the last condition clearly contradicts the infinitely thin layer mentioned by Kirchhoff.

4.3. Shadow of Black Body.

In the Kirchhoff definition of BB, it is said about the casting of a shadow by a black body. This statement implicitly says that the field of incident waves is spatially substantially anisotropic, i.e. is not a diffuse wave field (diffuse incident wave field arises in the problems of chamber acoustics with criteria of maximum absorbed power, see below Section 3.6). For a plane incident wave with a direction $\mathbf{w}$ of propagation, let us refine this formulation in terms of the scattering cross sections. In this case, the total scattering cross section $\sigma_{\mathrm{S}}(\mathbf{w})$ in all directions $\forall\mathbf{s}$ (meaning the scattering field in the far zone of the body) can be represented by two terms (see Fig. 3-g): the backward scattering cross section $\sigma(\mathbf{ws} < 0)$ (into the half-space $\mathbf{ws} < 0$) and the forward scattering cross section $\sigma(\mathbf{ws} > 0)$ (into the half-space $\mathbf{ws} > 0$). Moreover, for an ideal black body at $B/\lambda \gg 1$, the absorption cross section $\sigma_{\mathrm{A}}(\mathbf{w}) \approx \sigma(\mathbf{ws} > 0) \approx \sigma_{\mathrm{B}}(\mathbf{w})$is equal to the forward scattering cross section and the geometric cross section $\sigma_{\mathrm{B}}(\mathbf{w})$ of the black body (projection of the body onto the plane front of the incident wave, see Fig. 3-f), and the back scattering cross section is ideally equal to zero (or in practice $\sigma(\mathbf{ws} < 0) \ll \sigma_{\mathrm{B}}(\mathbf{w})$), this is there is a definition of "blackness" of the body. The forward scattering cross section $\sigma(\mathbf{ws} > 0)$ is not limited in this case (e $\forall\sigma(\mathbf{ws} > 0)$). Note that, for example, an attempt to minimize forward scattering $\sigma(\mathbf{ws} > 0) \to 0$ (with a free value $\sigma(\mathbf{ws} < 0)$) would inevitably entail minimizing backward scattering ($\sigma(\mathbf{ws} < 0) \to 0$) as well, i.e. this leads us to transparent body $\hat{B}$ at parameters constant in time. The backscattering cross section $\sigma(\mathbf{ws} < 0)$ is also called the body's effective scattering surface, and the value $\sigma(\mathbf{ws} < 0)/\sigma_B(\mathbf{w}) \ll 1$ characterizes the cloaking efficiency.

4.4. Black Body as a Source of Shadow

Let's make one more note about the casting of a shadow by the black body along the incident wave. Specifying the internal structure of a black body on the basis of its diffraction properties is an ill-posed problem and it has many options for solutions, including an active solution: from the point of view of an outside observer, a black body can be interpreted as some transparent (not reflecting source on the right side of the boundary conditions or wave equation) source $\hat{\mathrm{H}}$ of one-sided (only behind the body) radiation. Absorption of the incident wave can be interpreted as the performance of work by the incident wave. This work is applied to a one-sided radiation source $\hat{\mathrm{H}}$ (similar to the Huygens source, see Section 4.4.1. below). An incident wave with an amplitude $+A$ per unit time performs work $W > 0$ over the source $\hat{\mathrm{H}}$. In the process of performing this work (and oscillations of source $\hat{\mathrm{H}}$) scattered wave is generated with amplitude $-A$ and propagating (mainly) in the direction $\mathbf{w}$, and the power (dissipation power) carried by this wave to infinity is equal to W and this is close to the power that the incident plane wave carries in the geometric cross section σ_{B} of the body $\hat{\mathrm{B}}$. Waves generated by such a source $\hat{\mathrm{H}}$ against the background of an incident wave U_{w} could be called waves with negative energy, if this term had not already been attached to other problems. In this case, the energy of the incident wave can be converted not into heat, but into the energy of a source (a source of electromotive force in the case of electromagnetic waves) feeding a one-sided emitter $\hat{\mathrm{H}}$.

4.5. Models for External Problem of the Black Body.

The external problem of a black body is the approaches to solving the problem of diffraction of an incident plane wave by a black body. Despite Kirchhoff's idea that all the "blackness" of a black body is concentrated in an infinitely thin layer $\hat{\mathrm{L}}$ (literally, a layer, and not on the surface $\hat{\mathrm{S}}_{\mathrm{B}}$), all researchers when setting the problem of plane wave diffraction (with frequency ω and direction $\mathbf{w}$) by a black body only through the setting of boundary conditions (for simplicity for acoustical potential $U(\mathbf{r},\omega)$ on the frequency ω) of the form

$$U(\mathbf{r},\omega)\alpha(\mathbf{r}) + [\partial U(\mathbf{r},\omega)/\partial n]\beta(\mathbf{r}) = \gamma(\mathbf{r},\omega) \text{ for } \mathbf{r} \in \hat{\mathrm{S}}_{\mathrm{B}}, \quad (2)$$

where $\alpha(\mathbf{r})$, $\beta(\mathbf{r})$ are time-constant parameters on the black body $\hat{\mathrm{B}}$ surface $\mathbf{r} \in \hat{\mathrm{S}}_{\mathrm{B}}$, $\mathbf{n}$ is a unity ($|\mathbf{n}| = 1$) normal to $\hat{\mathrm{S}}_{\mathrm{B}}$ (the boundary condition with zero source $\gamma = 0$ in right-hand side is called *passive*, and the boundary condition with nonzero source $\gamma \neq 0$ in right-hand side is called *active*). At the same time, they assumed that wave processes in the region $\hat{\mathrm{B}}\backslash\hat{\mathrm{L}}$ (from the internal problem of the BB) cannot distort the need boundary condition (2). In addition we note that condition (2) can be applied only to waves without dispersion. Many attempts have been made by researchers to create the properties of a BB given by Kirchhoff's definition by varying the values $\alpha(\mathbf{r})$, $\beta(\mathbf{r})$, $\gamma(\mathbf{r},\omega)$, i.e. boundary conditions on the surface $\mathbf{r} \in \hat{\mathrm{S}}_{\mathrm{B}}$. Below, we will briefly describe three main directions (strategies) in solving the problem of diffraction on a BB (Fig. 4): (a) matching of the wave field or Kirchhoff's model[3]; (b) maximization of the absorbed power or Macdonald's model; (c) matching of the impedances or Weston's model. Each of these three models is the transfer of strategy of one-dimensional wave problem (with the same idea of reflectless entry into BB and with identical results) into a 2-3 dimensional wave problem (with different results, see Fig. 4). In a spatially one-dimensional problem, all three strategies just mentioned (a), (b), (c) give the same result: absorption coefficient $\tilde{A} = 1$, reflection coefficient $\tilde{R} = 0$, transmission coefficient $\tilde{T} = 0$. However, in 2, 3-dimensional problems, each of

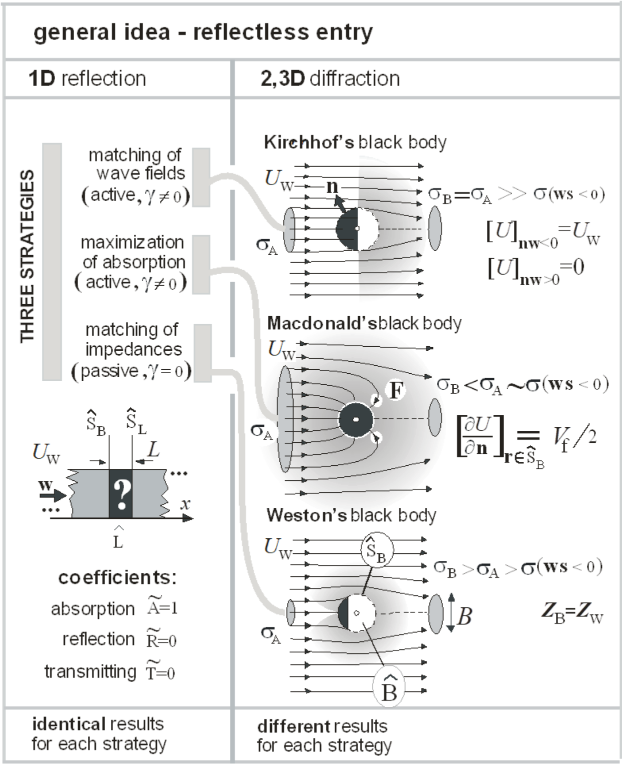

Fig. 4. On the genesis of three strategies for constructing a black body model from a one-dimensional problem to a three-dimensional one. Black on the surface $\hat{S}_B$ marks the area where the power flux lines reach the surface, and white - where they do not. The gray cloud marks the region of the near field (reactive field), which bends the power flux lines with the local vector $\mathbf{F}$. The boundary conditions arising as a result of each of the strategies are indicated.

the strategies lead to fundamentally different results. Now let us briefly outline the essence of three approaches to solving the external problem of blackbody.

Kirchhoff's method[3] (a) consists in the fact that on the illuminated (i.e., at $\mathbf{wn} < \mathbf{0}$, where $\mathbf{n}(\mathbf{r})$ is the outer normal to the surface $\hat{S}_B$) side of the surface $\hat{S}_B$, the field $U = U_w$ is equal to the field of the incident wave, and on the shadow side (at $\mathbf{wn} > 0$), the field $U = 0$ is zero. For a Kirchhoff's black body at an infinitesimal wavelength $\lambda \to 0$, the following relations $\sigma_B = \sigma_A \gg \sigma(\mathbf{ws} < 0) \to 0$ are satisfied for the blackbody absorption cross section σ_A, backscattering cross section $\sigma(\mathbf{ws} < 0)$, and the geometric cross section $\sigma_B(\mathbf{w})$ of the body itself. This boundary condition is active, since $\gamma/\alpha = U_w \neq 0$ at $\mathbf{wn} > 0$ and $\gamma/\alpha = 0$ at $\mathbf{wn} > 0$, $\beta = 0$ (boundary condition depends of $\mathbf{w}$ at any shape of $\hat{S}_B$).

MacDonald's method[7] consists in setting the normal velocity $V_f/2$ at the boundary, where $V_f = V_f(U_w)$ is the normal velocity arising from the scattering of the incident wave U_w at the free boundary $\hat{S}_B$ (i.e., with at the boundary with zero pressure or $U = 0$). This provides strictly $\sigma_A = \sigma(\mathbf{ws} < 0) + \sigma(\mathbf{ws} > 0)$ and the maximum absorbed power, but does not guarantee a small backscatter cross section $\sigma(\mathbf{ws} < 0)$. For smallness $\sigma(\mathbf{ws} < 0)$, it is necessary for the pattern $V_f(U_w)$ to radiate very weakly (in the absence of an incident wave U_w) in the directions $\mathbf{ws} < 0$, or, in other words, at $\sigma(\mathbf{ws} < 0, V_f(U_w)) \ll \sigma_B(\mathbf{w})$. This boundary condition is active, since $\gamma/\beta = V_f \neq 0$, $\alpha = 0$ (boundary condition depends of $\mathbf{w}$).even at any shape of $\hat{S}_B$). Only for the McDonald boundary condition is it possible $\mathbf{wF} < \mathbf{0}$, i.e. a reversal of the power flux lines (with local vector $\mathbf{F}$) on the back of the body $\hat{B}$. The disadvantage of masking under active boundary conditions is their dependence on the direction $\mathbf{w}$ of the incident wave, which gives rise to the dependence of the structure $\hat{\Xi} = \hat{\Xi}(\mathbf{r}, \mathbf{w})$ on $\mathbf{w}$. On the other hand, MacDonald's black body may not be effective in disguise, but an effective one in the task of reducing the rumble of the halls (Figs. 3-e, 3-f).

And, finally, the third way to suppress reflection from the boundary in a one-dimensional problem (where also the approaches of Kirchhoff and MacDonald originated) is to match (equalize) the boundary $\hat{S}_B$ impedance Z_B and the impedance Z_w of the incident wave. Weston[8] brings this approach to a 2-3 dimensional problem. Let P_s be the pressure of the scattering field of the incident wave on the body $\hat{B}$, and P_w the pressure of the incident wave. The normal speed of the boundary $\hat{S}_B$ according to Weston is $= P/Z_B$, where $P = P_w + P_s$ is the total sound pressure at the surface $\hat{S}_B$, Z_B local impedance of surface $\hat{S}_B$. The normal matched velocity of boundary consistent with the incident wave (i.e. the velocity at which there is no scattering or $P_s = 0$) of the boundary $\hat{S}_B$ is $V_0 = (\mathbf{nw})P_w/Z_w$., i.e. $V - V_0 \neq 0$ even at $Z_B = Z_w$. It is easy to verify that the boundary $\hat{S}_B$ oscillating at the Weston speed $V = P/Z_B$ in a 2-3-dimensional problem (as opposed to a one-dimensional problem) scatters the incident wave, i.e., strictly speaking; it does not provide a reflectionless entry of the wave into the black body. This causes the absorption cross section $\sigma_A < \sigma_B$ to be somewhat smaller than the body cross section σ_B, and, accordingly, some flow of power flux lines $\mathbf{F}$ around the body $\hat{B}$. Weston's boundary condition is passive, i.e. $\gamma(\mathbf{r}) = 0$, $\beta(\mathbf{r})/\alpha(\mathbf{r}) = Z_w = Z_B$, and it is indifferent to the direction of the incident wave (i.e. $\hat{\Xi} = \hat{\Xi}(\mathbf{r})$ at any shape of $\hat{S}_B$) and could serve as the basis for the development of a black shell (albeit not ideal in principle), if not for the lack of an answer to the question: how to convert wave energy into heat.

So for a monochromatic problem, the exact execution of a non-scattering input inevitably assumes (in 2D, 3D case and at parameters constant in time) the same output, i.e. body transparency leaving no room for absorption. For an exact solution (at parameters constant in time), only a choice is possible: either a transparent (without scattering) body without absorption, or absorption with considerable scattering at the input.

4.6. Attempts to implement the black body at parameters constant in time.

In this section, we will present a brief overview of attempts to solve the internal problem of a black body: creating an internal structure $\hat{\Xi}(\mathbf{r}, \omega)$($\omega$ -incident wave frequency) of a black body $\hat{B}$ that converts wave energy into heat with little backscattering at $\Re \ll 1$ and all conditions (1C)-(5C) at parameters constant in time, i.e. $\partial\hat{\Xi}/\partial t = 0$ and frequency ω^* of waves inside $\hat{L}$ is equal to the frequency $\omega^* = \omega$ of incident wave.

We already know from Kirchhoff's definition that all the functions of the black body $\hat{B}$ are performed by a thin shell $\hat{L}$. However, one cannot but dwell on the description of the pinhole camera as the

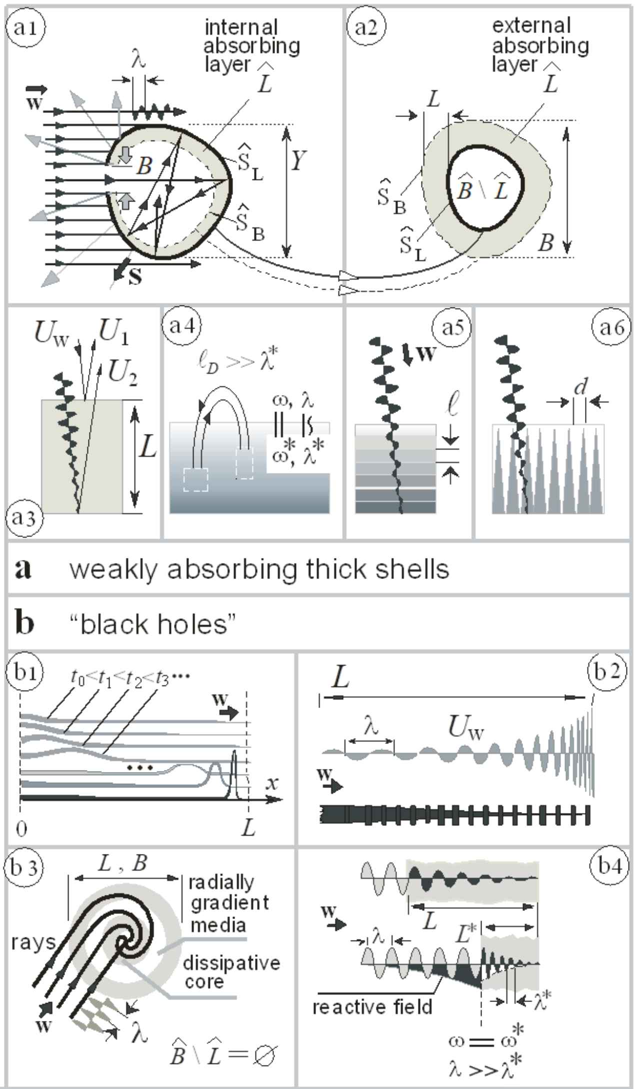

Fig. 5. (a) Practical versions of a black body based on the use of weakly absorbing media: (a1) pinhole camera; (a2) the pinhole camera turned inside out is a thick, weakly absorbing shell L̂; (a3) combination of smallness of reflection with effective absorption; (a4) gradient coating, small interaction of the absorbing elements of the shell at the frequency of the incident wave; (a5) many homogeneous layers; (a6) a set of dissipative wedge-pyramids. (b) Black holes instead of pinhole camera: (b1) time evolution of bends of a shepherd's whip (bending black hole); (b2) a notched bar as a vibrational black hole; (b3) a quasi-cosmological version of a black hole; (b4) matched entry into the medium with wavelength compression.

historically first and ingenious version of a black body, as well as the first version of a black hole. The pinhole camera (Fig. 5–a1) is a light-impenetrable (reflecting) box Ŷ (with an outer surface $\hat{S}_B$ and a linear dimension Y) with a cavity Ŷ\L̂ (with a surface $\hat{S}_L$ of a dissipative layer L̂ with a thickness $L \ll Y$) with a hole B̂ (a "black hole" with a size $B << Y$). Any light ray entering the hole B̂ is repeatedly reflected from the surface $\hat{S}_L$ (under which the dissipation is concentrated, i.e., the transformation of wave energy into heat) of the cavity and exponentially decreases its power with an increase in the number of reflections. The probability of the ray escaping (through the same hole B̂) from the pinhole camera trap is close to zero. Thus, the rays that hit the hole are not reflected, and the hole of the pinhole camera is actually a black body. The pinhole camera assumes beam optics ($Y \gg B \gg \lambda$) and does not contain a cavity with a zero field to accommodate the cloaked body D̂, which also does not meet the requirements (1C)-(5C) of cloaking presented in section 1.

Let's turn the pinhole camera Ŷ cavity Ŷ\L̂ inside out and cover the hole B̂ with a layer$c$. Now there will be a fully reflective surface $\hat{S}_L$ on the inside and a layer surface $\hat{S}_B$ on the outside (Fig. 5-a2). On the one hand, in order for the wave reflection at the entrance to the layer to be weak, it is necessary that the attenuation length ℓ_D (the length ℓ_D of the path on which the wave amplitude decreases by a factor $e \approx 2.7$ due to volume dissipation) in the layer L̂ medium should be significantly greater than the incident wavelength λ: $\ell_D \gg \lambda$. On the other hand, for a wave to be substantially absorbed in a weakly dissipative medium of a layer L̂ along a path with a length $2L$, the layer thickness L must be large enough: $2L \gg \ell_D$. The wave U_1 (Fig. 5-a3) of the reflection from the outer surface $\hat{S}_B$, and the wave U_2 completely reflected from the inner surface $\hat{S}_L$ and outward, attenuated by absorption on the path length $2L$ must both be small compared to the incident wave U_w, i.e. $(|U_1|/|U_w|) + (|U_2|/|U_w|) \ll 1$. Thus, the reflectionless input of the wave to L̂ is combined with effective absorption in L̂ due to the large thickness L and weak volume absorption (i.e $L \gg \ell_D \gg \lambda^*$) where λ^* is the wavelength in the shell L̂ medium (at $(\lambda - \lambda^*)/\lambda \approx \lambda^*/\ell_D \ll 1$) and, in other words, the weak interaction of the absorbing elements of the shell at the frequency of the incident wave (Fig. 5-a4).

Dissipative cladding is a known way to darken window panes. In practice, such a shell is simple, reliable, broadband, but too thick and heavy (for aviation for instance). Further work was associated with attempts to create a thinner (and at the same time lighter) broadband absorbing coating without growth $\Re$ (see Section 1). These attempts made it possible to reduce the thickness of the shell to a value $L{\sim}2\lambda$), thanks to the use of the following techniques: (1) Smooth variation of the dissipation parameter (dissipation length ℓ_D) with depth (from $\ell_D = \infty$ to $\ell_D = L/2$) the penetration of the incident wave into the shell L̂ (Fig. 5-a4). Disadvantages: weight, dimensions, expensive technology of a shell smooth in depth; (2) The shell L̂ is an echelon of $N \gg 1$ thin (with a thickness $\ell = L^*/N$) homogeneous layers with a decreasing in depth length of dissipation in the material of the layers (Fig. 5-a5).

Disadvantages: weight, dimensions, many types of materials with different dissipation lengths; (3) Since the reflection from a flat layer with strong dissipation is large, it is possible to replace the echelon of thin layers with a grating (Fig. 5-a6) of conical wedges (or pyramids pointing outward) from a material with strong dissipation. The wedges create a layer L̂ effect with smooth dissipation variation as in the approach described above, but with a single type of homogeneous absorbent material (the air-wedge structure is light, simple and cheap). One drawback remains - an unacceptably large shell thickness L for $\lambda = \lambda_{max}$.

Relatively recently, a famous line of research on "black holes" as black body models in relation to electromagnetic and acoustic waves has emerged. This made it possible to view the black body as an object absorbing waves, having some relation to cosmology, and also to find a new application for the Wentzel - Kramers - Brillouin (WKB) mathematical apparatus. This direction included the design of the structure $\hat{\Xi}(\mathbf{r})$ of the absorbing shell $\hat{\mathrm{L}}$ and the calculation of the corresponding diffraction field. Due to the term "black hole", it was possible to extend the study of systems with time-constant parameters and even re-consider spatially one-dimensional problems as one-dimensional black holes. Naturally, about a one-dimensional black hole, the shepherd's whip is remembered as the first vibrational black hole (Fig. 5–b1). Such a black hole (Fig. 5–b2) implies the following physical moments: (1) reflectionless entry into the system; (2) a gradual decrease in the group velocity of the wave as the layer $\hat{\mathrm{L}}$ deepens; (3) an increase in the amplitude of the wave as it stops; (4) extremely fast dissipative attenuation of the wave in the vicinity of the "stopping point" of any recoil to the reflected wave (for a shepherd's lash, dissipative absorption means air breakdown when the speed of the whip tip passes the threshold of sound speed into air). And shepherd does not feel by hand any signal about this loud click. There are also known constructions of "black holes",[12] using the movement of rays in the same way as the movement of material particles in the gravitational field (there no cavity $\hat{\mathrm{B}}\backslash\hat{\mathrm{L}} = \emptyset$ for body $\hat{\mathrm{D}}$ to be protected, Fig. 5–b3). Naturally, all of these black hole models cannot satisfy masking conditions (1C)-(5C) (Section 1).

According to the article,[13] it is possible, at constant parameters of the shell $\hat{\mathrm{L}}$, to transform the wavelength λ^* in the shell $\hat{\mathrm{L}}$ abruptly $\lambda \to \lambda^* \ll \lambda$ (with a new shell $\hat{\mathrm{L}}$ thickness $L^* \ll L$) with respect to the incident wavelength λ without reflection at the boundary $\hat{\mathrm{S}}_{\mathrm{B}}$ (Fig. 5-b4), but using the anisotropy of the shell parameters and allowing the reactive field (pressed to the border). However, anisotropy inevitably entails wave dispersion, which is a strong obstacle to broadband absorption. In addition, for a non-spherical shell $\hat{\mathrm{L}}$ and a nonplanar incident wave, the structure $\hat{\Xi}(\mathbf{r}, \omega, \mathbf{w})$ should depend on the direction $\mathbf{w}$ of the incident wave and its frequency ω, i.e. here all the same difficulties arise as for perfect cloaking[2] in Section 2.

5. Parametric Version of Black Body

In this section, we will consider the possibility of using high spatial-temporal resolution tools inside the shell $\hat{\mathrm{L}}$ to simultaneously fulfil all conditions (1C)-(5C) masking (section 1) of the black shell. With constant time parameters of the PT, it was never possible to jointly fulfil the masking requirements (1C)-(5C):

1. To absorb a wave, it is necessary to provide it with the opportunity to perform work (convert the wave energy into heat) on the absorbing device, and to perform this work, the wave need a spatial interval $\geq \lambda$ and a time interval $\geq \lambda/c$. Therefore, it was not possible to provide effective non-resonant absorption in a layer of small wave thickness.
2. In monochromatic problems with time-constant parameters and traditional boundary conditions given on the surface of the body, it is not possible to model (without internal contradictions) diffraction on a black body.
3. In systems with time-constant parameters, an obstacle in providing broadband effective masking is the inevitable interaction of absorbing elements at the frequency of the incident wave. Therefore, at constant parameters of the system in time, thick($L \gg \lambda$) weakly absorbing coatings remain practically effective, where weak local volume dissipation means a small interaction of absorbing elements.

In parallel with attempts to overcome the aforementioned difficulties 1–3, microelectronics, aimed at computer applications, developed for a long time and very successfully. According to the empirical law of Gordon Moore, the minimum spatial $L_{//}$ (dimensions) and temporal (speed) τ_{sw} scales of microelectronic devices are halved every two years (today it is already slower, but still over the past 50 years, tremendous successes have been achieved), i.e.,

$$\tau_{\mathrm{SW}}\,(\text{today})/\tau_{\mathrm{SW}}\,(2\text{ years ago}) = \mathrm{L}_{//}\,(\text{today}) \,/\, \mathrm{L}_{//}(2\text{ years ago}) \approx 1/2$$

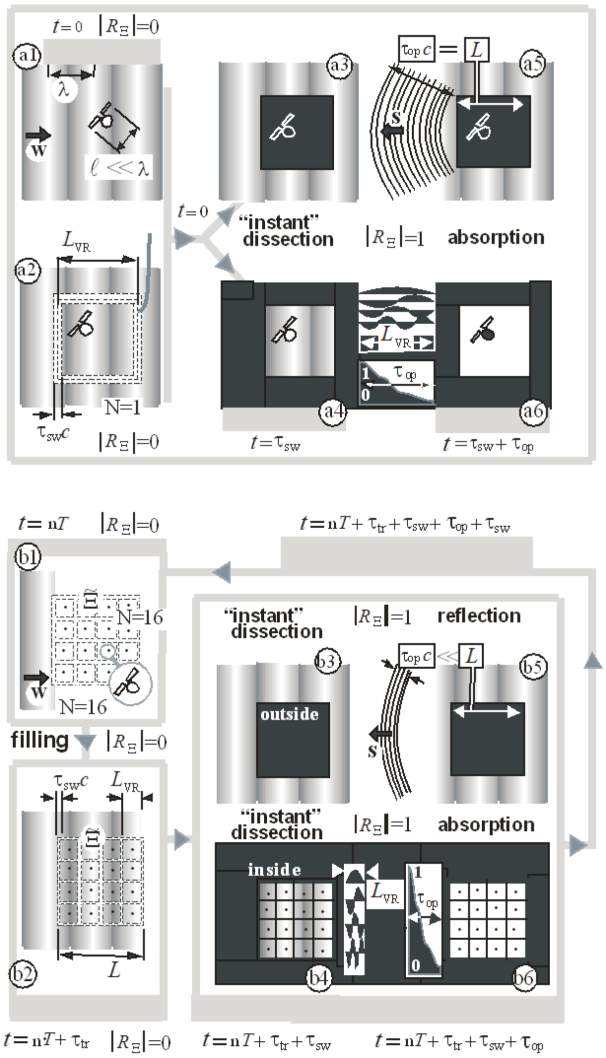

Fig. 6. **(a)** Instant single simultaneous dissection ((a1), (a2)) of space along the contour of a single virtual resonator and separation of the boundary value problem into external ((a3), (a5)) and internal ((a4), (a6)); (b) periodic (with a temporal period $T = \tau_{\mathrm{tr}} + \tau_{\mathrm{op}} + 2\tau_{\mathrm{sw}}$ and spatial period $L_{\mathrm{VR}} \ll \lambda$) instantaneous simultaneous dissection of space by the contour $\hat{\Xi}(\mathrm{r})$ and absorption of the field that hit the resonators with a decrease in its energy from level "1" to level "0" in time $\boldsymbol{\tau}_{\mathbf{op}} \ll L/c$.

At the same time, the lengths of the absorbed incident waves all these years remained unchanged due to the preservation of the natural conditions of their distant propagation. The text presented be-low is an attempt to satisfy the Kirchhoff definition of a blackbody (in particular, to find ways to create an absorbing shell $\hat{\mathrm{L}}$ with a thickness $L \ll \lambda$), based on the operations of controlling the boundary conditions of the boundary value problem with a very high spatial-temporal resolution $\lambda/L_{//} \gg 1$, $\lambda/(c\tau_{SW} / 2\pi) \gg 1$ ($L_{//}$ – minimum space scale of elements of shell $\hat{\mathrm{L}}$) (see [14,15]). Along the way with this task (fulfilment of conditions (1c) and (5C)), conditions (2C)-(4C) are also fulfilled. Suppose that for this purposes we can on the minimal spatial interval $L_{//} \ll L$, for a very short time $\tau_{\mathrm{sw}} \ll L_{//}/ c$, perform a deep modulation of the parameters of the medium (or boundary conditions on its elements) in the region $\hat{\mathrm{L}}$. To describe the modulation depth of parameters (for simplicity, in an acoustic problem), let us formulate a certain scalar parametric boundary condition for the field $U(\mathbf{r},t)$ on an arbitrary section $\mathbf{r} \in \hat{S}\hat{L}$ of the surface of some microelement $\hat{\mathrm{L}}$ of the shell (which we are going to make black): $[\alpha U'_n(\mathbf{r},t) + \beta U'_t(\mathbf{r},t)]_{\mathbf{r}\epsilon\hat{S}\hat{L}} = 0$, where $\alpha(\mathbf{r},t)$, $\beta(\mathbf{r},t)$ are time-varying coefficients (parameters similar to the boundary condition (2) in section 3.5). Deep modulation means the fulfillment of the condition $1-[(b-a]/(b+a)] \ll 1$, where $a = min|\beta(\mathbf{r},t)/\alpha(\mathbf{r},t)|$, $b = max|\beta(\mathbf{r},t)/\alpha(\mathbf{r},t)|$. In the acoustic case, this means switching the boundary of the "(soft) - (hard)" type, and in the electromagnetic case, for example, switching the boundary of the "(metal) - (vacuum)" type. Usually (in contrast to the case considered in this work), modulation of parameters, for example, in parametric resonances, means weak modulation when the condition $[(b-a]/(b+a)] \ll 1$ is satisfied. The effect of such modulation accumulates to large values over many periods of oscillation. In addition, there is a widespread judgment that a problem with fast modulation of parameters (without synchronization with an external stimulus) can be described as a problem with constant parameters equal to the time average. In the case considered in this article, such a judgment is not applicable.

5.1. The way to solve the Problem

In a three-dimensional (as opposed to one-dimensional) problem with parameters constant in time, a non-scattering entry into the body is impossible without the complete exit of all the entered wave energy from the body. Therefore, we will have to (controlling in time the parameters of the shell $\hat{\mathrm{L}}$ structure $\hat{\Xi} = \hat{\Xi}(\mathbf{r},t)$ to separate in time the process of admission (or propagation) of the field and the process of absorption of the field (transformation of the field into heat). Thus, if we begin to alternate with a period $T = L/c$ in time the processes of field propagation inward $\hat{\mathrm{L}}$ (of duration $\tau_{\mathrm{tr}} = T - \tau_{\mathrm{op}}$) and the process (of duration $\tau_{\mathrm{op}} \ll \tau_{\mathrm{tr}}$) of complete absorption of the field inside $\hat{\mathrm{L}}$ (naturally, in the process of such intense absorption, the shell $\hat{\mathrm{L}}$ cannot be transparent, so we will consider it completely opaque), then Kirchhoff's definition of a black body (Section 3.1) and conditions (1C)-(5C) (Section 1) the disguises will be performed, since the modulus $|\Gamma|$ of the reflection coefficient from such a structure in the one-dimensional case will have an order of magnitude at any small thickness. Now there is only one question

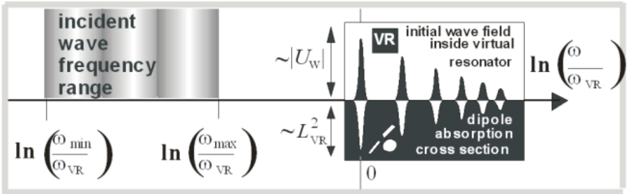

Fig. 7. The hierarchy of the scale of the boundary value problem: the location on the logarithmic axis of the frequencies (normalized to the lowest natural frequency $\omega_{\mathrm{VR}} = \pi c/L_{\mathrm{VR}}$ of the virtual resonator) of the frequency range $\omega \in \hat{\omega}[\omega_{\min} \le \omega \le \omega_{\max}]$ of the incident waves, the natural frequencies of the virtual resonator, and the frequency distribution of the resonant absorption cross section $\sim L^2_{VR}$ of a specially tuned dipole (dipoles).

left: how to ensure full absorption of the field in a shell $\hat{\mathrm{L}}$ with a thickness $L \ll \lambda$ during the time $\tau_{\mathrm{op}} \ll L/c$?

Let us imagine some foamy 3-dimensional structure $\hat{\Xi}(\mathbf{r},t)$ with infinitely thin walls with the modulus $|R_\Xi(\mathrm{t})|$ of the reflection coefficient (as applied to the one-dimensional problem) located along the contour $\hat{\Xi}(\mathbf{r})$ (Fig. 6).

We will call the single-connected regions (with a linear scale $L_{VR} \ll L$) inside $\hat{\Xi}(\mathbf{r})$ virtual resonators: when $|R_\Xi(\mathrm{t})| = 0$ there is no resonator, but $|R_\Xi(\mathrm{t})| = 1$ when it is. Inside each such region there is an absorbing dipole (with a size $\ell \ll L_{VR}$) tuned to the maximum absorption (with an absorption cross section $\sim L^2_{VR}$) at discrete natural frequencies $\omega_1, \omega_2, \omega_3$,… of the given resonator (see Fig. 7). The zero natural frequency $\omega_0 = 0$ in such resonators with binary ($|R_\Xi(\mathrm{t})| = (0 \leftrightarrow 1)$) modulation of the reflection coefficient is absent, and the minimum nonzero frequency $\omega_1 \sim \pi c / L_{VR} \gg \omega_{max}$ is so high that when $|R_\Xi(\mathrm{t})| = 0$ the incident wave freely (without scattering) passes through the array of high-frequency dipoles. With instant (in time $\tau_{\mathrm{sw}} \ll \tau_{\mathrm{op}}$) simultaneous dissection (switching on $|R_\Xi(\mathrm{t})| = 1$) of space along the foam like contour $\hat{\Xi}(\mathbf{r})$ of the space regions inside the virtual resonators, they become completely independent of each other.

Thus, in contrast to the systems considered in Sections 2 and 3 with parameters constant in time, the absorbing dipoles in the structure $\hat{\Xi}(\mathbf{r},t)$ do not interact with each other at the frequency of the incident wave, which has always been the cause of the dependence $\hat{\Xi}(\mathbf{r},t)$ on ω and $\mathbf{w}$, i.e. $\hat{\Xi}(\mathbf{r},\omega,\mathbf{w})$.

Fundamentally important is the fact that when the opacity mode ($|R_\Xi(\mathrm{t})| = 1$) is quickly turned on, almost all points (except for a strip with a thickness $\sim\tau_{sw}c \ll L_{VR}$ along the walls) inside the virtual resonator "do not have time to know" about what happened on the contour $\hat{\Xi}(\mathrm{r})$. Therefore, the field at these points continues to run as in an incident wave. Therefore, the instantaneous spatial distribution (of the order of magnitude, see Fig. 7) of the wave field within one virtual resonator becomes the initial conditions for damping (due to the dipole) oscillations inside the resonator. In addition, an arbitrarily small (with size $L_{\mathrm{VR}} \ll \lambda$) fragment of the incident traveling wave. These oscillations decay the faster, the higher the first natural frequency $\omega_1 \sim \pi c / L_{VR}$ and the smaller L_{VR}. These oscillations can be damped over time $\tau_{\mathrm{op}} \ll T$ if the shell $\hat{L}$ is divided (along the normal to its surface) into many small resonators, thereby reducing the time $\tau_{\mathrm{op}} \ll L/c$ (and averaged on period $T = \tau_{\mathrm{tr}} + \tau_{\mathrm{op}} + 2\tau_{\mathrm{sw}}$

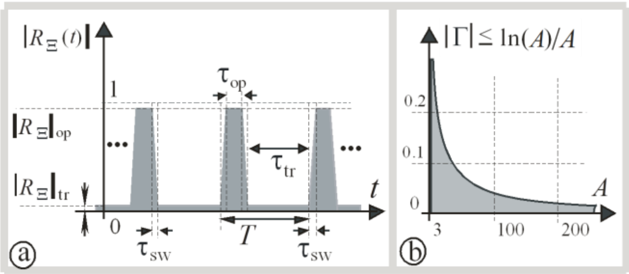

Fig. 8. Cyclic wave bolt in a one-dimensional problem: (a) plots of the modulus of the reflection coefficient of the walls, (b) estimation of the modulus (mean over the period T) of the reflection coefficient from the one-dimensional structure $\hat{\Xi}(\mathbf{r},t)$ with ideally transparent ($|R_{\Xi}|_{tr} = 0$) and ideally opaque ($|R_{\Xi}|_{op} = 1$) reflection coefficients and instantaneous switching ($\tau_{sw} = 0$).

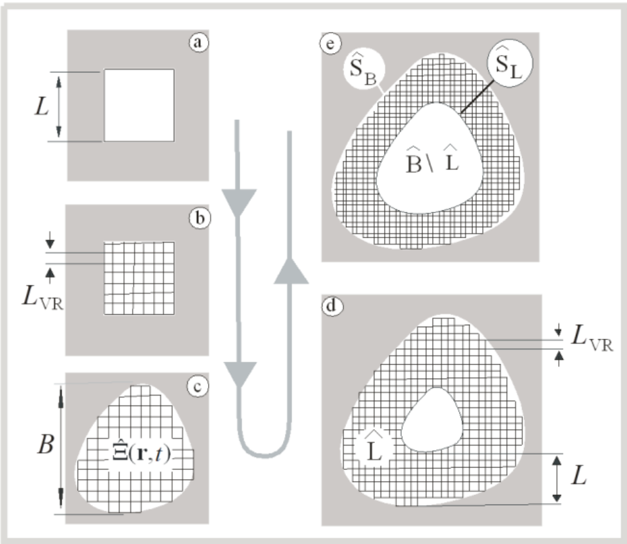

Fig. 9. Microstructuring ($L_{VR}/B \to 0$) and evolution of the foam-like structure $\hat{\Xi}(\mathbf{r},t)$ (or contour $\hat{\Xi}(\mathbf{r})$) from a single virtual resonator (a), to an array of virtual resonators similar to a single one (b), to an array of arbitrary shape (c), to an array of arbitrary shape with a cavity $\hat{B}\backslash\hat{L}$ inside $\hat{B}$ (d), to an increase in the cavity with a decrease in the shell $\hat{L}$ thickness L without an increase in the relative cross section $\Re = \sigma(\text{ws} < 0)/\sigma_B \ll 1$ of backscattering of the structure $\hat{\Xi}$ for external ($\mathbf{r} \notin \hat{B}$) sources of incident waves (e).

intensity) of external reflection of the incident wave by the structure during periodic (with a period T, where, see Fig. 8-a) functioning of the structure as shown in Fig. 6-b.

This was necessary to separate in time the processes of propagation and the absorption process (or, in other words, to combine the reflectionless entry of waves into the black body required in Kirchhoff's definition and their complete absorption in it).

To estimate roughly the incident wave scattering by parametric black body we consider briefly a one-dimensional analog of structure $\hat{\Xi}(\mathbf{r},t)$ with contour $\hat{\Xi}(\mathbf{r})$ (cyclical wave-bolt [14,15]), which is a train of walls of controlled transparency i.e., with reflection coefficient $R_{\Xi}(t)$ which are equidistant (with a spatial period L_{VR}) located in the path of the incident wave. Any two adjacent walls of controlled transparency are a virtual resonator with length L_{VR} and own frequencies $\ge c\pi/L_{VR}$. Fig. 8-a, shows the modulus $|R_{\Xi}(t)|$ of the reflection coefficient of one (each) wall as a function of time. The upper estimate of optimal (minimum) average over the period T of the modulus of the reflection coefficient (Fig. 8-b) gives the value $|\Gamma| \le 1n(A)/A$, at $\tau_{op}/\tau_{tr} = 1n(A)/A \ll 1$, where $A = (M_x/M_t)\,\alpha$, $M_t = T/\tau_{op} \gg 1$ is the microstructuring coefficient in time, $M_x = L/L_{VR} \gg 1$ is the microstructuring coefficient in space, $\boldsymbol{\alpha}$ is the decay damping factor of a wave over a single path across the resonator (until we specify the damping mechanism (wave absorption by walls, leakage high-frequency field components through the resonator walls, wave absorption in the cavity environment, wave absorption by resonator dipoles, etc.).

Fig. 9 shows the conversion of one virtual resonator to black shell $\hat{L}$ with arbitrarily small thickness due to the microstructuring the shell $\hat{L}$. Virtual resonators in a contour $\hat{\Xi}(\mathbf{r})$ can have an arbitrary convex shape (not only square) and dimensions L_{VR} varying from one to other in space.

5.2. One Version of Walls of Controlled Transparency

In this section, we will consider a possible arrangement of walls with variable transparency [on the parametric version]. The metal grid (Fig. 10-a) is the equivalent of the metal plane. This is very important for the point about the absence of zero natural frequency for the resonator formed by metal grids. This property of a mesh woven from metal wires is used for parabolic reflectors in radars. Having accepted $\lambda = L_{VR}/2$, we will obtain (for $L_{\#} < \lambda/4$) an estimate $\left|1 - |R_{\Xi}|_{op}\right| \approx 2(L_{\#}/L_{VR})|1(L_0\pi/L_{\#}| \ll 1$ of the modulus of the reflection coefficient $|R_{\Xi}|_{op}$ of the wall of the structure $\hat{\Xi}(\mathbf{r},t)$ in an opaque state. Now let's think about how to make the wire mesh transparent. Consider an infinite single (Fig. 10–b) ideally conducting wire of infinite length and radius, on which a plane linearly polarized electromagnetic wave with amplitudes $\mathbf{E}_I$ and $\mathbf{H}_I$ of both electric and magnetic fields, respectively, and a propagation vector $\mathbf{w}$, is normally incident. The ratio of the modulus $|J_{\#0}|$ of current in one wire of a periodic (with period $L_{\#}$) lattice (Fig. 10-c) wires to the modulus $|J_0|$ (Fig. 10-b) of current in a single wire (generated by a linearly polarized incident wave with amplitudes $\mathbf{E}_w$ and $\mathbf{H}_w$ of both electric and magnetic fields, wavelength λ and direction $\mathbf{w}$) can be estimated as $|J_{\#0}|/\ |J_0| \approx (\lambda/L_{\#})/|1n(L_0\pi/\lambda)| \gg 1$ at (when the lattice of wires becomes equivalent to the metal plane and the current J_0 is equal to the electric current through a part of the metal plane in the form of a tape with a width $L_{\#}/\lambda < 1/4$, Fig. 10-c).

Such a large difference in the amplitudes of the currents is due to the presence of electrostatic interaction between the wires in the lattice (the angular size of the near wire from the side of the neighbouring one is equal $= \pi$). It is enough to cut the wires in the lattice into pieces of length $L_{//}$ (with the corresponding parameter $\vartheta \le L_{//}/L_{\#} \ll 1$ of electrostatic interaction) in order to further evaluate the current $J_{\#1}$ (Fig. 10-d) in one of the wires of lattice of splitted wires as the current $J_1 \approx J_{\#1}$ (Fig. 10-e), which is excited by the incident wave in a single splitted wire. Next, insert into the cuts of the wires (Fig. 10-e) electronic switches with a through capacity C_{sw} (really $C_{sw} \sim 10^{-12} F$). Thus, the reflection coefficient from such a "transparent" (when the switches are closed and do not con-duct current, Fig. 10-f) lattice of cut wires, we estimate as $|R_{\Xi}|_{tr} \le (L_{//}/L_{\#}) C_{sw}\omega_{max}\sqrt{\epsilon_0/\mu_0} \ll 1$, where ϵ_0 and μ_0 are the electric and

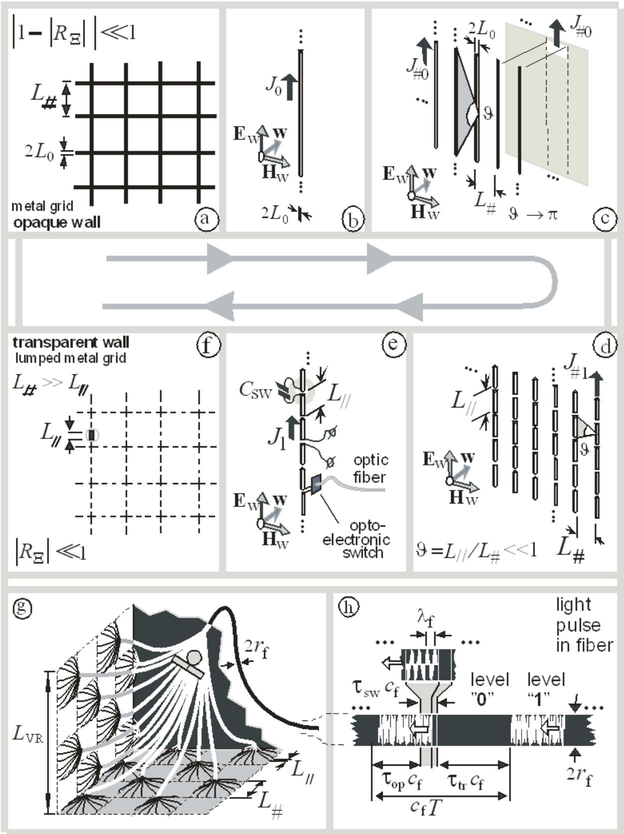

Fig. 10. How to create a wall of controlled transparency: (a) a grid of uncut wires as equivalent to a metal plane (radar reflectors); (b) current J_0 in a single uncut wire; (c) an array of parallel wires (with currents $J_{\#0}$) as equivalent to a metal plane; (d) the array of the cut wires and the smallness factor ϑ of the electrostatic interaction of the parallel cut wires and the current $J_{\#1}$ in each wire; (e) a single wire with cuts into which optoelectronic switches are inserted with a through capacity C_{sw} and a current $J_1 \approx J_{\#1}$ (at $\vartheta \ll 1$) in it; (f) transparent wall - a metal grid with cuts (i.e. with closed optoelectronic switches and light intensity "0" in the fiber), and vice versa (a) opaque wall (a) is a grid with open optoelectronic switches in cuts and light intensity "1" in fiber; (g) a virtual resonator with an absorbing dipole and walls, the transparency of which is determined by the light intensity in the optical fiber, the optical length of all fibers (taking into account branching) is the same for simultaneous switching on of all optoelectronic switches; (h) moving of laser pulses along the optical fiber.

magnetic vacuum constants, respectively, $\boldsymbol{\omega}_{max}$ is the maximum frequency of the incident wave. A rigorous solution to the problem of the transparency for metal strip structure is given in Cohn S.B.[16] The above considered walls of controlled transparency form the main functional element of the black shell - a virtual resonator (Fig. 10–g). Metal wires of arbitrarily small thickness and length are capable of creating an undesirable stray field with a scattering cross-section size much larger than the geometric dimensions of the conductor (for example, a metal grid - a reflector). Therefore, for switching the transparent and opaque states of the walls, galvanically independent miniature high-speed optoelectronic switches controlled by signals from optical fibers were chosen. For the simultaneous on-off for optoelectronic switches, the lengths of the optical fibers (taking into account branching) must be the same. The optoelectronic switch reacts to the average (during switching time about τ_{sw}~10^{-10}s) light intensity in the optical fiber. Therefore, to change the state of the key at the right time, it is necessary: many periods of the light wave and a single-mode mode in an optical fiber. For a certain characteristic wavelength λ_F of laser light in a fiber (for example, $\lambda_F = 6.5\,X\,10^{-7}$m, Fig. 10-h), the condition of a lot of periods of a light wave on the switching interval is necessary (switching occurs after a change in the time average laser intensity "$0 \longrightarrow 1$" or "$1 \longrightarrow 0$") $c_F\tau_{sw} \gg \lambda_F$ (where c_F is the speed of light in the fiber) and the conditions of multiple wavelengths of light on the fiber radius (single mode in the fiber) $r_F/\lambda_F > 3$. Glass parts create a field of scattering proportional to only the relative average volume of the glass (relative to the air volume of the resonator $\sim L_{VR}^3$ in the ideal). To power the optoelectronic switches, you can also use the time average level of light intensity in the fiber. However, it is necessary to require that the relative volume of glass in the resonator should be small, i.e.$(\Delta V/V)_F \approx 12\pi\, L_F^2/(L_\# L_{VR}) \ll 1$, where $r_F \approx 3.5X\,10^{-6}$m - characteristic radius of one-mode optic fiber. All fibers inside the resonator have the same length so that the switches should be triggered simultaneously. Now we require the smallness of relative total volume of all smallest metal details (of length $L_{//}$ and radius L_0) in VR, i.e. the condition: $(\Delta V/V)_M 12\pi\, L_0^2/(L_\# L_{VR}) \ll 1$. The considered model uses ultra-fast operations with metal pieces. We assume that it is possible to set (during the time much smaller then τ_{sw}) classical boundary condition on the metal surface (zero of the tangential component of the electric field). But at arbitrarily high switching frequencies, the properties of the electrons of a metal as a gas may enter the game. Therefore, we need to formulate the conditions under which the switch would represent a sequential change in time of the static charge distributions on the metal surface. At 300^0 Kelvin, the characteristic Maxwell's relaxation time of some perturbed charge distribution in the metal to the equilibrium distribution is $\tau_M = 1.25 \times 10^{-15}$ s, and the plasma wavelength in the metal is $\lambda_P \approx 4 \times 10^{-7}$m. Thus, to ensure the static nature of processes in metal elements with a length $L_{//}$, the following conditions must be met: $\tau_{sw} \gg \tau_M$, $L_{//} \gg \lambda_P$.

5.3. Algorithm for solving the problem of diffraction by a parametric black body

For a strict (non-compromise) fulfilment of the Kirchhoff blackbody definition, it is necessary to eliminate the fundamental (for systems with constant parameters) contradiction between the reflectionless entry of all waves into the layer $\hat{\mathrm{L}}$ and the total absorption in $\hat{\mathrm{L}}$ all of the same waves crossed from outside to inside the surface $\hat{S}_B$. Using the cyclic wave gate algorithm (see Fig. 8) or controlling the transparency (reflectance coefficient R_Ξ(t)) of the walls of the foam-like layer $\hat{\mathrm{L}}$ structure $\hat{\Xi}(\mathbf{r},t)$, we separate two processes in time: (1) the process of wave propagation from region $\mathbf{r} \notin \hat{B}$ to region $\hat{\mathrm{L}}$ (in time intervals of duration $\tau_{tr} = L/c$), when the field penetrates into the layer $\hat{\mathrm{L}}$ no deeper than to its inner surface $\hat{S}_{\mathrm{L}}$, without meet any scattering obstacles; (2) the process of absorbing the field or converting it into heat in loads (the loads are resonantly tuned to the natural frequencies of the virtual resonator (see Fig. 10), among which there is no zero frequency, and the smallest nonzero frequency is of the order $\omega \geq \pi c\,/L_{VR} \gg \omega_{max}$) of dipoles inside virtual resonators in intervals of duration $\tau_{op} \gg \tau_{tr}$. At $\tau_{tr} \to 0$

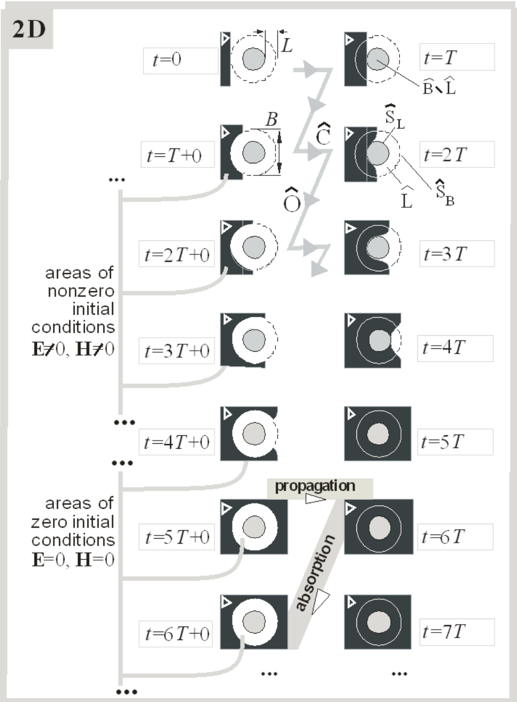

Fig. 11. Illustration of an algorithm (sequence of operations) for solving the problem of diffraction of a plane wave (black region) falling from the left onto a black box $\hat{\mathrm{L}}$ (or black shell) of cylindrical shape with outer $\hat{S}_B$ and inner $\hat{S}_{\mathrm{L}}$ surfaces and thickness L.

(at $L/L_{VR} \to 0$) the process of diffraction of the incident wave by an ideal parametric black body can be represented as a sequence (with a period $= L/c$) of solutions of the Cauchy problem[17] on the field propagation from the area $\mathbf{r} \notin \hat{\mathrm{B}}$ (at $R_{\Xi} = 0$) separated from each other by arbitrarily short time intervals (at $|R_{\Xi}|$ =1) absorption of the wave field inside $\hat{\mathrm{L}}$ and, respectively, instantaneous (ideally) zeroing of the field at points $\mathbf{r} \notin \hat{\mathrm{L}}$, as shown in Fig. 11. Note that at the beginning of each new time interval of the shell $\hat{\mathrm{L}}$ transparency, the field outside $\hat{\mathrm{L}}$ ($\mathbf{r} \notin \hat{\mathrm{L}}$) is nonzero ($E, H \neq 0$), and inside $\hat{\mathrm{L}}$ ($\mathrm{r} \notin \hat{\mathrm{L}}$) the field is zero ($E, H \neq 0$). Therefore, the field can only propagate in one direction: from outside $\hat{\mathrm{L}}$ to inside $\hat{\mathrm{L}}$ (as required by the definition of the blackbody of Kirchhoff). Thus, due to the separation in time (by means of commutation in time of the shell $\hat{\mathrm{L}}$ parameters) of the intervals of field propagation and its absorption (transformation into heat), we managed to combine the transparency of a black body at the entrance of a wave into it, prescribed in Kirchhoff's definition, and the total absorption inside the black body. It was impossible when trying to set the boundary conditions (2) (values $\alpha(\mathbf{r})$, $\beta(\mathbf{r})$ unvariable in time and values $\lambda(\mathbf{r},\omega)$ variable in time) on the black body surface (see section 3.5. above). For a linear boundary value problem with constant nonsingular coefficients of the hyperbolic wave equation, one can construct (at least numerically) some linear operator (Cauchy operator $\hat{C}$ ={$\hat{C}_{EE}$, $\hat{C}_{HE}$, $\hat{C}_{EH}$, $\hat{C}_{HH}$}, integral over space) that solves the Cauchy problem on the evolution (propagation) of the spatial distribution of the field in time from the moment $t = t_0$ to the moment $t = t_0 + T$ (provided that in a given time interval $t_0 \le t \le t_0 + T$ the shell $\hat{\mathrm{L}}$ is transparent and its parameters are constant in time): $\boldsymbol{E}(\mathbf{r}, t_0 + T) = \hat{C}_{EE}\, \boldsymbol{E}(\mathbf{r}, t_0) + \hat{C}_{EH}\, \boldsymbol{H}(\mathbf{r}, t_0)$ and $\boldsymbol{H}(\mathbf{r}, t_0 + T) = \hat{C}_{HE}\, \boldsymbol{E}(\mathbf{r}, t_0) + \hat{C}_{HH}\, \boldsymbol{H}(\mathbf{r}, t_0)$ ($\boldsymbol{E}$ is the strength of the electric field, $\boldsymbol{H}$ is the strength of the magnetic field).

To describe the results of the process of instantaneous (over time $\tau_{op} \ll T$) absorption, we will use the operator $\hat{\mathrm{O}}$ of zeroing the field in the area in the time interval $t \in [t_0, t_0 + 0]$:

$\boldsymbol{E}(\mathbf{r}, t_0 + 0) = \hat{O}[\boldsymbol{E}(\mathbf{r}, t_0)] = 0$ for $\mathbf{r} \in \hat{L}$,
$\boldsymbol{E}(\mathbf{r}, t_0 + 0) = \hat{O}[\boldsymbol{E}(\mathbf{r}, t_0)] = \boldsymbol{E}(\mathbf{r}, t_0)$ for $\mathbf{r} \notin \hat{L}$,
$\boldsymbol{H}(\mathbf{r}, t_0 + 0) = \hat{O}[\boldsymbol{H}(\mathbf{r}, t_0)] = 0$ *for* $\mathbf{r} \in \hat{L}$,
$\boldsymbol{H}(\mathbf{r}, t_0 + 0) = \hat{O}[\boldsymbol{H}(\mathbf{r}, t_0)] = \boldsymbol{H}(\mathbf{r}, t_0)$ for $\mathbf{r} \notin \hat{L}$.

The speed (instantaneousness) of the field zeroing means that the points $\mathbf{r} \notin \hat{\mathrm{L}}$ do not have time to know about the field zeroing at points $\mathbf{r} \notin \hat{\mathrm{L}}$, with the exception of points $\mathbf{r} \notin \hat{\mathrm{L}}$ remote from the surfaces $\hat{S}_{\mathrm{B}}$, $\hat{S}_{\mathrm{L}}$ no more than a distance $\tau_{op} c \ll L$ that we can make much less than the layer $\hat{\mathrm{L}}$ thickness L .

Now let us formulate a step-by-step algorithm for solving the diffraction problem of a parametric black body. Let for all $\mathbf{r}$ be given an arbitrary initial spatial distributions of fields $\mathbf{E}(\mathbf{r}, \mathrm{t}_0)$ and $\mathbf{H}(\mathbf{r}, \mathrm{t}_0)$ at the moment $t = t_0$.

Step 1: we cut out (zero out, absorb) the fields $\mathbf{E}$ and $\mathbf{H}$, in the region $\hat{\mathrm{L}}$: $\overline{\boldsymbol{E}}(\mathbf{r}, \mathrm{t}_0) = \hat{\mathrm{O}}[\mathbf{E}(\mathbf{r}, \mathrm{t}_0)], \overline{\boldsymbol{H}}(\mathbf{r}, \mathrm{t}_0) = \hat{\mathrm{O}}[\mathbf{H}(\mathbf{r}, \mathrm{t}_0)]$.

Step 2: fields $\overline{\boldsymbol{E}}(\mathbf{r}, \mathrm{t}_0)$ and $\overline{\boldsymbol{H}}(\mathbf{r}, \mathrm{t}_0)$ propagate inside the transparent shell (becoming the initial conditions of the Cauchy problem) and in time T are transformed to the form

$\mathbf{E}(\mathbf{r}, \mathrm{t}_0 + T) = \hat{\mathrm{C}}_{\mathrm{EE}}\overline{\boldsymbol{E}}(\mathbf{r}, \mathrm{t}_0) + \hat{\mathrm{C}}_{\mathrm{EH}}\overline{\boldsymbol{H}}(\mathbf{r}, \mathrm{t}_0)$ and
$\mathbf{H}(\mathbf{r}, \mathrm{t}_0 + T) = \hat{\mathrm{C}}_{\mathrm{HE}}\overline{\boldsymbol{E}}(\mathbf{r}, \mathrm{t}_0) + \hat{\mathrm{C}}_{\mathrm{HH}}\overline{\boldsymbol{H}}(\mathbf{r}, \mathrm{t}_0)$.

Step 3: $\overline{\boldsymbol{E}}(\mathbf{r}, \mathrm{t}_0 + T) = \hat{\mathrm{O}}[\mathbf{E}(\mathbf{r}, \mathrm{t}_0 + T)]$.
$\overline{\boldsymbol{H}}(\mathbf{r}, \mathrm{t}_0 + T) = \hat{\mathrm{O}}[\mathbf{H}(\mathbf{r}, \mathrm{t}_0 + T)]$.

Step 4: $(\mathbf{r}, \mathrm{t}_0 + 2T) = \hat{\mathrm{C}}_{\mathrm{EE}}\overline{\boldsymbol{E}}(\mathbf{r}, \mathrm{t}_0 + T) + \hat{\mathrm{C}}_{\mathrm{EH}}\overline{\boldsymbol{H}}(\mathbf{r}, \mathrm{t}_0 + T)$,
$\mathbf{H}(\mathbf{r}, \mathrm{t}_0 + 2T) = \hat{\mathrm{C}}_{\mathrm{HE}}\overline{\boldsymbol{E}}(\mathbf{r}, \mathrm{t}_0 + T) + \hat{\mathrm{C}}_{\mathrm{HH}}\overline{\boldsymbol{H}}(\mathbf{r}, \mathrm{t}_0 + T)$.

Step 5: $\overline{\boldsymbol{E}}(\mathbf{r}, \mathrm{t}_0 + 2T) = \hat{\mathrm{O}}[\mathbf{E}(\mathbf{r}, \mathrm{t}_0 + 2T)]$.
$\overline{\boldsymbol{H}}(\mathbf{r}, \mathrm{t}_0 + 2T) = \hat{\mathrm{O}}[\mathbf{H}(\mathbf{r}, \mathrm{t}_0 + 2T)]$.

Step 6: $\mathbf{E}(\mathbf{r}, \mathrm{t}_0 + 3T) = \hat{\mathrm{C}}_{\mathrm{EE}}\overline{\boldsymbol{E}}(\mathbf{r}, \mathrm{t}_0 + 2T) + \hat{\mathrm{C}}_{\mathrm{EH}}\overline{\boldsymbol{H}}(\mathbf{r}, \mathrm{t}_0 + 2T)$,
$\mathbf{H}(\mathbf{r}, t_0 + 3T) = \hat{\mathrm{C}}_{\mathrm{HE}}\overline{\boldsymbol{E}}(\mathbf{r}, t_0 + 2T) + \hat{\mathrm{C}}_{\mathrm{HH}}\overline{\boldsymbol{H}}(\mathbf{r}, t_0 + 2T)$.

…etc. The above algorithm for solving the problem of diffraction by parametric black body is universal: any shape of a body, any wave dimensions of body, any shape of an incident wave front. However, the difficulty is in a much larger amount of computation than at traditional approaches (as in Section 3.5.). Fig. 11 represents the just described sequence of operations of a parametric (cylindrical) black shell and the leading edge of a plane incident wave: the area of non-zero field is shown in black and gradually (periodically) fills the white area of the zero field. Such a procedure for solving the diffraction problem can be called the "diffraction of the incident wave on periodically updated initial conditions" (unlike the traditional "diffraction on boundary conditions").

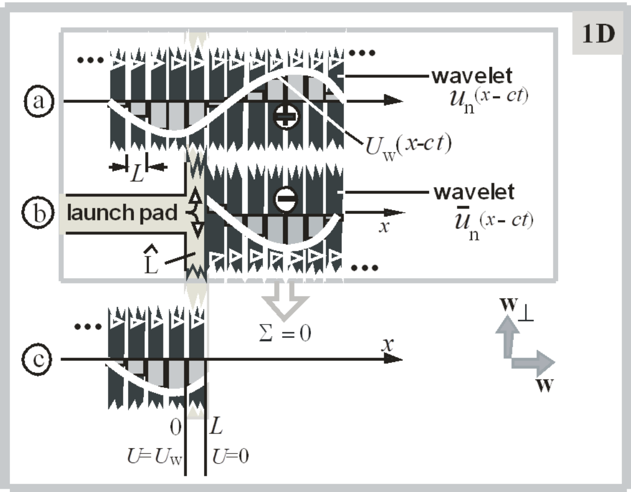

Fig. 12. (a) Representation of a one-dimensional plane smooth running to the right incident wave U_w by a sequence $\Sigma_n u_n$ of plane one-dimensional traveling wavelets u_n; (b) instantaneous absorption generates anti-wavelets $\bar{u}_n = -u_n$ at the launch pad; (c) the sum of the wavelet fields u_n and $\bar{u}_n$.

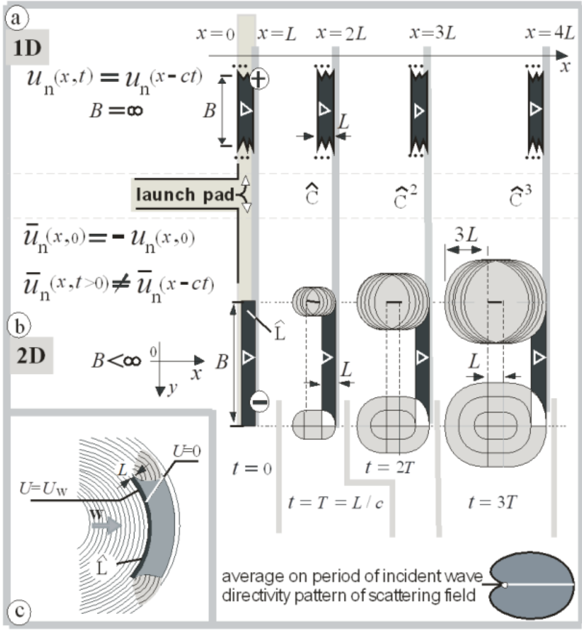

Fig. 13. (a) Propagation of a one-dimensional single flat (with transverse length $B = \infty$ and longitudinal length L) wavelet $u_n(x-ct)$; (b) Propagation and evolution of an anti-wavelet $\bar{u}_n(x,t)$ (the black color indicates the area where the traveling field is one-dimensional, the gray color indicates the expanding region of the nonzero field of edge waves; (c) Parametric thin black screen $\hat{L}$ (thick black line) conformal to a spherical incident wave satisfying Kirchhoff's boundary conditions.

5.4. Flat Thin Parametric Black Screen

The classical scenario (considered by Kirchhoff himself) in the study of diffraction by a black body $\hat{B}$ is the diffraction of a plane wave at normal incidence on a thin flat black screen $\hat{L}$. We emphasize: in this case, a thin (thick) flat black screen is a black body without a cavity (i.e. $\hat{B}\backslash\hat{L} = \emptyset$) and with a finite transverse dimension $B \gg L$. Now we consider the diffraction of a plane incident wave on a plane black parametric screen. Let's imagine a plane (for example, electromagnetic, linearly polarized) smooth (on distance) incident wave with the electric field strength $U_w(\mathbf{rw}-ct) = U_w(x-ct)$ the sum $U_w(x-ct) \approx \Sigma_n u_n(x-ct)$ (where $n = 0,1,2,,\dots$, $u_n(x-ct) = U_w(nL)\bar{I}(x-ct-nL)$ $\bar{I}(\xi) = I(\xi) - I(\xi-L)$, $I(\xi) = 1$ at, at $\xi > 0$, $I(\xi) = 0$ at $\xi < 0$) plane non-intersecting homogeneous in the transverse directions rectangular wavelets $u_n(x-ct)$ (let's call them so) of length L traveling in the direction $\mathbf{w}$.

Such impulses run with speed c along axis "x" , without changing their structure in the longitudinal $\mathbf{w}$ and transverse $\mathbf{w}_\perp$ directions ($\partial/\partial\mathbf{w}_\perp = 0, |\mathbf{w}_\perp| = 1, \mathbf{w}_\perp\mathbf{w} = 0$, Fig. 12–a) without crossing and without gaps between adjacent wavelets. This is a property of the wave equation with separable variables. However, provided that transverse width $B < \infty$ of pulse is finite, such impulse diffuses over time in space, starting from their edges (where). We note: such a representation is possible only for non-dispersive waves; each of the wavelets can be instantly removed (absorbed) without any effect on the movement and shape of the remaining wavelets. Suppose that at some moment $t = nT$ the left and right boundaries of the wavelet $u_n(x-ct)$ have coincide, respectively, with the left and right boundaries of the plane layer $\hat{L}(0 < x < L)$. At the moment $= nT$, the instantaneous absorption of the field in the layer $\hat{L}$ is switched on. Now the action of an infinite flat black parametric black screen $\hat{L}$ on the wave field can be expressed by periodic (with a period $T = L/c$) instantaneous absorption of the field in the layer $\hat{L}$. At the moment $t = nT$ immediately after the instantaneous absorption of the incident wavelet $u_n(x-ct)$, the energy of the scattered field is accumulated in the initial condition $-u_n(x-cnT)$ $(0 < x < L)$, which generates at $t > nT$ the wavelet $\bar{u}_n(x-ct) = -u_n(x-ct)$, (Fig. 12-b) compensating the field of the incident wavelet at $x > L$ (Fig. 12-c). A pulse of inverted polarity is formed during $\tau_{op} \ll \tau_{tr}$ in the process of absorption of an incident pulse that has entered the region $\hat{L}$. It is enough to create in the area $\hat{L}$ (on the launch pad $0 < x < L$) a field of a traveling pulse (i.e., a special space distribution of fields $\mathbf{E}(\mathbf{r}, t_0)$ and $\mathbf{H}(\mathbf{r}, t_0)$) for one instant $t = t_0$ and then this pulse will run, carrying away the final energy (in the form of wavelet $\bar{u}_n(x-ct)$) spent on its creation and equal to the absorbed energy of the pulse $u_n(x-cnT)$. In our case, the tool for creating the distribution $-u_n(x-cnT)(0 < x < L)$ is the instantaneous absorption of the pulse $u_n(x-cnT)$ from the wavelet sequence $\Sigma_n u_n(x-ct)$ of the incident wave $U_w(x-ct)$. Let us turn to the non-one-dimensional case (in particular, to the two-dimensional case, see Fig. 13). A wavelet u_n infinite in the direction $w_\perp$ (a component of the incident wave) propagates (shifts to the right, see Fig. 13-a) without changes, i.e.$[\hat{C}]^m u_n(x-ct) = u_n(x-ct-mL)$, where $= 0,1,2,\dots$, $\hat{C}$ the Cauchy operator of field evolution in time T.

In Fig. 13-b shows roughly a time evolution of a wavelet $\bar{u}_n(x-ct)$ bounded in the direction $\boldsymbol{w}_\perp$ by size B ($\infty > B \gg L$). The area marked in black "knows nothing" about edge waves and continues to move (and transfer energy) as an infinite (i.e. as at $B = \infty$) wavelet (Fig. 13-a). The growing region of a nonzero edge wave is shown in gray. The time-averaged (and averaged in area $\hat{L}$) relative contribution ε (when the cancelling wavelet $\bar{u}_n(x-ct)$ generated by the parametric screen $\hat{L}$ periodically restores the uniform (along $\boldsymbol{w}_\perp$) field distribution, and the edge diffusion wave

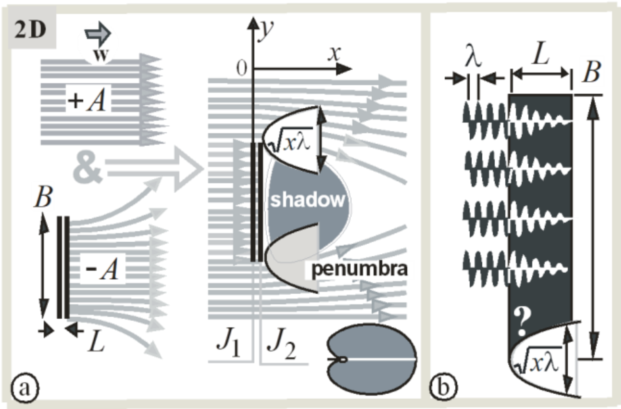

Fig. 14. The flat Huygens source as a shadow source and the equivalent of a thin black screen: (a) 2D problem, power flux lines of plane incident wave (with complex amplitude $+A$), of Huygens source (generating the wave with amplitude $-A$) and for this source in the incident wave field; (b) incompatibility of passive (dissipative) absorption and screen of thickness $L \ll \lambda$.

decreases as $\sim|\bar{u}_n|(t/T)^{-2}$ of the edge waves can be estimated as the sum $\varepsilon \leq (L/B)\sum_{n=1}^{\infty} n^{-2} = (\mathrm{L/B})\zeta(2)$, where $\zeta(2) \approx 1.2$ the Riemann zeta function of argument 2. Thus $\varepsilon \to 0$, at $/\mathrm{B} \to 0$, and absorption cross section $\sigma_A = \sigma_B$ (average on the incident wave period $2\pi c/\lambda$) is equal to geometrical cross section $\sigma_B \sim D^2$ for the flat thin black screen. This means that, on average, over the period $T = L/c$ of the cyclic wave shutter, the Kirchhoff boundary conditions are satisfied for a thin black flat screen: $\lim_{L/D\to 0;y\epsilon[-B/2,+B/2]} U(0,y,t) - U_{\mathrm{w}}(0,y,t) = 0$ on the illuminated side of the screen and $\lim_{L/D\to 0;y\epsilon[-B/2,+B/2]} U(L,y,t) = 0$ on its back. Similarly, one can show the fulfilment of the Kirchhoff boundary conditions in the diffraction problem on a thin black conformal parametric screen (see Fig. 13-c), which completely repeats the incident wave front (when one of the surfaces of constant phase passes through a given point of the screen).

5.4.1. The flat Huygens source as a shadow source and the equivalent of a thin black screen

Now we add the consideration of active (non-dissipative) flat thin black screen. We consider two spatially one-dimensional problems for monochromatic electromagnetic waves: (a) a plane linearly polarized electric field wave with a complex amplitude A (along the axis "y") propagates from left to right (in direction $\mathbf{w}$ along the axis "x"); (b) in free space (perpendicular to $\mathbf{w}$) two planes with currents $J_1 = -J(-A)\exp(-ikL)/[\exp(ikL) - 1]$ (to the left at the point $x = 0$) and $J_2 = J(-A)/[\exp(ikL) - 1]$ (to the right at the point $x = L$) are located at a distance L from each other (the distance L can be arbitrarily small ($L \ll \lambda$), but with a greater amplitude of currents $|J_1|, |J_2| \gg |J(-A)|$), where $J(-A)$ is the complex amplitude of the flat current (at a point $x = 0$) which is required to create a linearly polarized wave with a complex amplitude $-A$ of the electric field. Such a system (Huygens source), consisting of two currents (not metals with currents, but only currents or flows of charges in a vacuum), is transparent to waves, does not radiate to the left, and to the right this system emits a wave that completely compensates incident wave if $B = \infty$. Now let's make the transverse (along the "y" coordinate) size of the boundary value problem with a finite scale $B < \infty$ (see Fig. 14-a), and as a result we will get a shadow source. It is easy to see that such a Huygens source (source of shadow) is a thin (possible $\ll \lambda$) flat black screen. The width of the penumbra increases as $\sim\sqrt{x\lambda}$ with the distance x from the Huygens source edge in the direction $\mathbf{w}$. Directly behind the source, the field is equal to zero, and immediately ahead of the Huygens source, the field is equal to the field of the incident wave, as required by the Kirchhoff approach (Section 3.5.) if $L \ll B$. i.e. there the Kirchhoff boundary conditions are satisfied (as above for parametric black screen):

$$\lim_{L/D\to 0;y\epsilon[-B/2,\ +B/2]} U(0,y,\omega) - U_w(0,y,\omega) = 0,$$
$$\lim_{L/D\to 0;y\epsilon[-B/2,\ +B/2]} U(L,y,\omega) = 0.$$

On the other hand, the passive (dissipative, see Fig. 14-b) realization of a thin black screen needs condition $L \ll \lambda$ (not a thin) and can't satisfy Kirchhoff's boundary condition due to significant penumbra field along thickness $L \ll \lambda$. This shows incorrectness of traditional statement of the problem with dissipation in the screen with parameters constant in time.

6. Conclusions

In the presented article, the fundamental difficulties of designing and describing a black body with constant parameters (as well as in a monochromatic representation) are formulated: (a) thin ($L \ll \lambda$) shell is not possible because to transform a wave into heat (the internal problem of a black body), the wave needs space $\geq \lambda$ and time $\geq \lambda/c$ to make the work with the absorbing element; (b) the difficulty of creating broadband absorption is caused by the interaction of the absorbing elements at the frequency of the incident wave; (c) the impossibility of a rigorous description of diffraction on a black body (external problem of a black body) using any boundary conditions on its surface is due to the fact that strict fulfillment of the reflectless entry of the incident wave into the black body (with constant parameters and at monochromatic representation) inevitably means the same unhindered exit from the body, i.e. not a black body, but a transparent body. It is shown that the above three difficulties can be overcome by the parametric version of the black body, described in this article, designed for waves without dispersion. The internal structure $\hat{\Xi}(\mathbf{r},t)$ of the parametric black shell $\hat{\mathrm{L}}$ (internal blackbody problem) is a foam-like contour $\tilde{\Xi}$ with flat walls of controlled transparency or reflection coefficient $R_\Xi(t)$. In the parametric version of the black body, the deepest binary (switching $R_\Xi = (0 \Leftrightarrow 1)$) modulation of the shell parameters is used at the minimum space-time scales of modulation (traditionally, monochromatic shallow parametric modulation with a slow accumulation of the effect in time was considered) and is qualitatively characterized by the expressions "need to be in time", "need to have time", "need to have enough time to make"... In this case, the space of the parametric shell is periodically dissected in time by a foam-like opaque $R_\Xi = 1$ contour $\tilde{\Xi}$ into simply connected areas (virtual resonators) so quickly (in time $\tau_{\mathrm{sw}} \to 0$) that almost all points of these fragments (except for points at a distance $< \tau_{\mathrm{sw}} c \to$

0 from the dissecting contour $\tilde{\Xi}$) do not have time to do find out about the dissection that occurred and their complete isolation from the rest of the space. A wave field that suddenly finds itself inside simply connected spatial fragments becomes the initial conditions for oscillations at the Eigen frequencies of virtual resonators. Since virtual resonators do not have a zero natural frequency (this is how the walls of the cutting contour $\tilde{\Xi}$ are made), the lowest natural frequency is determined by its geometrical dimensions $\sim L_{\mathrm{VR}}$ as $\pi c/L_{\mathrm{VR}} \gg \omega_{max}$. An absorbing dipole located inside each virtual resonator (at $R_{\Xi} = 0$ the dipole is insensitive to a low-frequency incident wave), have been tuned to very high natural frequencies of the virtual resonator, and within a time $\sim(2L_{\mathrm{VR}}/c)Q$ (where $Q > 1$ is the Q-factor of the resonator in the presence of an absorbing dipole) during several periods has time to convert the energy of the resonator field into heat. By reducing the size of virtual resonators (with dimensions $L_{\mathrm{VR}} \ll L$), we can make the damping time of oscillations in them much shorter than the free travel time L/c of the incident wave through the shell $\hat{\mathrm{L}}$ thickness L in the e transparent state (at $R_{\Xi} = 0$). The latter circumstance makes it possible to combine the free entry of a portion (duration L/c and length L) of the external field into $\hat{\mathrm{L}}$ and its "instantaneous total absorption" inside $\hat{\mathrm{L}}$. Thus, a parametric blackbody periodically (with a period $T \approx L/c \ll \omega_{max}$) absorbs the energy of the external field in portions $\sim W_{\mathrm{w}} B^2 T$, where W_{w} is the time-average power flux density of the external field U_{w}. Due to the separation in time (as opposed to systems with time-constant parameters) in the parametric version of the black body there is no interaction of the absorbing elements (dipoles inside virtual resonators) at the frequency of the incident wave. Therefore the parametric version of the black body is free from unwanted selectivity for frequency and direction of the incident wave. The lower frequency of the incident wave is not limited by anything (which is impossible for a body with parameters constant in time). And the upper frequency of the incident wave, due to the device of the parametric layer, is obviously higher than all frequencies of the incident waves used for long-range propagation. Despite the simplicity and linearity of the parametric blackbody idea, this solution cannot be obtained by synthesizing any monochromatic solutions (when $\partial/\partial t = i\omega$). Before this solution, one can only "guess". The boundary value problems corresponding to the transparent and opaque states of the walls of the foam-like dissecting contour (structure $\hat{\Xi}(\mathbf{r}, t)$ of layer $\hat{\mathrm{L}}$) cannot be connected analytically, since their connection requires an infinite (or with a width $\sim 2\pi/\tau_{\mathrm{sw}} \gg \varpi$) Fourier integral, and any sinusoidal component taken separately will be devoid of physical sense. A large number of figures in the article are due to the difficulties in describing the boundary value problem by mathematical formulas at the temporal representation of the problem. Only separate blocks of the boundary value problem can be described by formulas: reflection from a structure, attenuation of waves in virtual resonators, field propagation in a transparent state of the structure $\hat{\Xi}(\mathbf{r}, t)$.

A specific (Section 4.2) parametric version of the blackbody is based on the binary control of the conductivity of optoelectronic switches. Conductivity is a dissipative parameter of the electromagnetic field equations. Therefore, such control cannot generate any instability of the structure $\hat{\Xi}(\mathbf{r}, t)$.

Thus, the parametric version of the black body with an arbitrarily small (infinitely small) thickness $L \ll \lambda$ of the black shell $\hat{\mathrm{L}}$ satisfies the Kirchhoff definition and ensures: (a) the fulfillment of conditions (1C)-(5C) of masking (Section 1); (b) solving the internal (converting the incident wave into heat) blackbody problem; (c) solution of the external (diffraction of the incident wave by a black body) black body problem.

It is shown that the Kirchhoff boundary conditions on a thin flat black screen (when there is no cavity $\hat{\mathrm{B}} \backslash \hat{\mathrm{L}} = \emptyset$) are accurate for a parametric black screen (Section 4.4) and for its active version (Section 4.4.1). At the same time, it is shown that, in the general case (in 2D-3D problems and at $\hat{\mathrm{B}} \backslash \hat{\mathrm{L}} \neq \emptyset$), it is impossible to set boundary conditions on a black body (there can be no boundary conditions on the surface of a black body) without violating any assumptions formulated in Kirchhoff's definition.

It is surprising that Kirchhoff's definition of a black body speaks of an infinitely thin (or arbitrarily thin) absorbing layer (or shell) that solves the external and internal problems of the black body. Perhaps it was the result of Kirchhoff's amazing physical, esthetic or stylistic intuition. But, may be, Gustav Kirchhoff himself had already considered a parametric blackbody (the mathematical foundations of this approach were known to him), but perhaps he did not consider it possible to publish because it might seem too far from 19th century.

Supporting Information

This work was supported by the Presidium RAS Program No.5: Pho-tonic technologies in probing inhomogeneous media and biological objects.

Conflicts of Interest

The authors declare no conflict of interest.

References

1 Xiaojun Qiu; Haishan Zhou. Recent Progress in Research on Virtual Sound Barriers. Proceedings of ACOUSTICS 2016 9-11 November 2016, Brisbane, Australia. [Link]

2 Cummer S.A.; Popa B.I.; Schurig D.; Smith D.R.; Pendry J.; Rahm M.; Starr A. Scattering Theory Derivation 3D Acoustic Cloaking Shell. *Phys. Rev. Lett*., 2008, **100**. [CrossRef]

3 Kirchhoff G. Mathematische Optik (Vorlesungen uber Mathematischer Physik, Band 2). Leipzig: Druck und Verlag von B.G. Trubner, 1891. 80 s. [Link]

4 Balanis C.A. *Antenna Theory: Analysis and Design*. John wiley & sons, 2015. [Link]

5 Black Body. [Link]

6 Planck M. The Theory of Heat Radiation. Masius, M. (transl.) (2nd ed.) P. Blakiston's Son & Co. OL 7154661M, 1914, 10. [Link]

7 Macdonald H.M. X. The Effect Produced by an Obstacle on a Train of Electric Waves. *Philosophical Transactions of the Royal Society of London. Series A, Containing Papers of a Mathematical or Physical Character*, 1913, **212**, 299-337. [CrossRef]

8 Weston V. Theory of Absorbers in Scattering. *IEEE Transactions on Antennas and Propagation*, 1963, **11**, 578-584. [CrossRef]

9 Delfini A.; Albano M.; Vricella A.; Santoni F.; Rubini G.; Pastore R.; Marchetti M. Advanced Radar Absorbing Ceramic-based Materials for Multifunctional Applications in Space Environment. *Materials*, 2018, **11**, 1730. [CrossRef]

10 Mitsmaher M.Yu.; Torgovanov V.A. Microwave Anechoic Chambers, M. Radio I Sviaz, 1982. (in Russian). [Link]
11 Mironov M.A. A Notched Bar as a Vibrational Black Hole. *Acoust. Phys.*, 2019, **65**, 634-636. [CrossRef]
12 Cheng Q.; Cui T.J.; Jiang W.X.; Cai B.G. An Electromagnetic Black Hole Made of Metamaterials. *arXiv preprint arXiv:0910.2159,* 2009. [Link]
13 Yefimov Sergei P. Absolutely Black-body in Diffraction Theory. *Radio Eng. Electron. Phys*., 1978, **23**, 6–13. [Link]
14 Arabadzhi V.V. *Solutions to Problems of Controlling Long Waves with the Help of Micro-Structure Tools*. Bentham Science Publishers, 2011. [Link]
15 Arabadzhi V. On the Parametric Version of Black Body. *Nano Tech. Appl.*, 2019, **2**, 1-6. [Link]
16 Cohn S.B. Analysis of the Metal-Strip Delay Structure for Microwave Lenses. *J. Appl. Phys.*, 1949, **20**, 257-262. [CrossRef]
17 Hadamard J.; Philip M. Morse. Lectures on Cauchy's Problem in Linear Partial Differential Equations. *Phys. Today*, 1953, **6**, 18. [CrossRef]